TANJONG KATONG SECONDARY SCHOOL

Preliminary Examination 2024
Secondary 4

CANDIDATE NAME	
CLASS	INDEX NUMBER

MATHEMATICS

4052/01

Paper 1

Wednesday 14 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Curved surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Simplify $\left(\frac{3}{w^2}\right)^{-4}$.

Answer ..... [1]

(b) Show that $\frac{2^{3x+4}-11\times 2^{3x}}{5\times 2^{x}}$ is a multiple of 4 where x is a positive integer.

.....

.....[3]

---

2 The expression $x^2-12x-8$ is equivalent to $(x-a)^2-b$.

(a) Find the value of a and the value of b.

Answer a = ... [1]

b = ... [1]

(b) State the coordinates of the turning point of the graph of $y=x^2-12x-8$.

Answer (..... , ...) [1]

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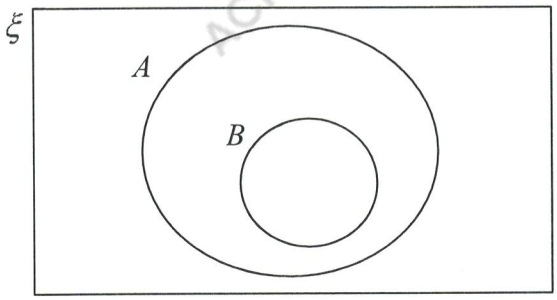
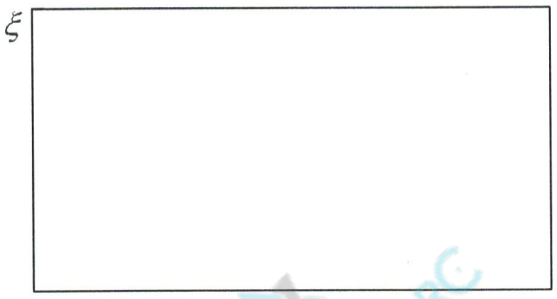
3 **(a)** On the Venn diagram, shade the region which represents $A \cap B'$. [1]

Answer



(b) Draw a Venn diagram of the three sets X, Y and Z such that $X \cap Y = \varnothing$ and $(X \cup Y) \subset Z$. [1]

Answer



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4 Written as a product of its prime factors,

$14\ 580 = 2^2 \times 3^6 \times 5$

$50\ 000 = 2^4 \times 5^5$

(a) Find the greatest integer that divides both 14 580 and 50 000.

Answer ..... [1]

(b) Using prime factors, explain why $14\ 580 \times 50\ 000$ is both a perfect square and a perfect cube.

.....

.....[1]

(c) m and n are both prime numbers.

Find the value of m and of n so that $50\ 000 \times \frac{m}{n}$ is a perfect cube.

Answer m = ... [1]

n = ... [1]

---

5 John is planning a business trip made up of two legs. He starts in Singapore and first travels to Europe, where he exchanges 400 Singapore dollars ($) to euros (€). After spending €50 in Europe, he exchanges the remaining euros for Japanese yen (¥) for his next stop in Japan.

The exchange rates at the money changer are as follows:
€1 = $1.50
¥100 = €0.74

Calculate the amount of Japanese yen John has for his next stop in Japan.
Leave your answer to the nearest yen.

Answer ¥..... [3]

---

6 Factorise completely.

(a) $a^2 - 2ap + 2pq - aq$,

Answer ..... [2]

(b) $u^2 - (v^2 + w^2 + 2vw)$.

Answer ..... [2]

---

7 An alloy A contains copper and tin in the ratio 2 : 3 by weight.

(a) Find the weight of copper, in grams, in 1 kg of alloy A.

Answer .....g [1]

Another alloy B contains tin and zinc in the ratio 5 : 7 by weight.

(b) Find the ratio of copper, tin and zinc in a new alloy containing **equal** weights of alloys A and B.

Answer ..... : ..... : ..... [2]

8 **(a)** Rajesh invests $4200 in a bank that pays compound interest of 2.4% per annum, compounded every half-yearly. Find the amount of interest he earns after 30 months.

Answer $..... [2]

(b) A laptop has a cash price of $2888.
A buyer can purchase it by making a 15% deposit followed by 12 equal monthly instalments of $250.

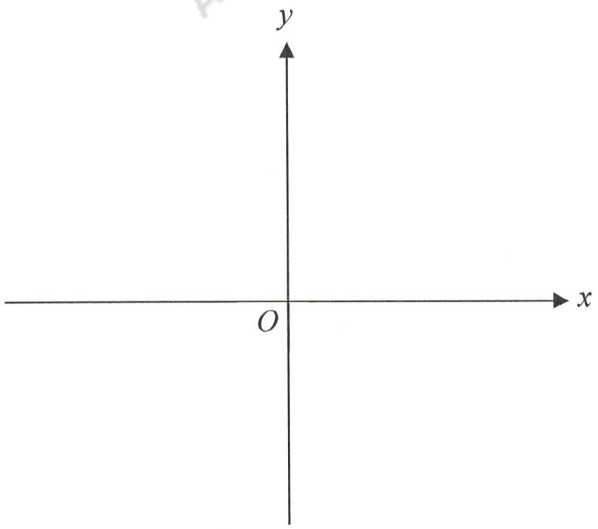
(i) Calculate the hire purchase price.

Answer $..... [2]

(ii) Calculate the percentage interest per annum on the outstanding balance under the hire purchase scheme.

Answer .....% [3]

9 **(a)** Sketch the graph of $y = (x+1)(3-x)$ on the axes below.
Indicate clearly the points where the graph crosses the axes. [2]



(b) Find the equation of the line of symmetry of the curve $y = (x+1)(3-x)$.

Answer ..... [1]

---

10 **(a)** In 2023, the average number of working hours in a day is 8 hours and 30 minutes.
Find, in hours and minutes, the average number of working hours in a day in 2022 if it was 10% less than that in 2023.

Answer ..... [2]

(b) x litres of Solution A containing 74% acid is to be combined with 5 litres of Solution B containing 90% acid in order to obtain a mixture containing 84% acid.
Form an equation in x and solve it.

Answer $x =$ ..... [3]

---

11 Amos, Betty and Carol took a multiple choice test containing 40 questions.

Amos has 28 correct answers, 8 incorrect answers and did not attempt 4 questions.
Betty has 32 correct answers, 7 incorrect answers and did not attempt 1 question.
Carol has x correct answers and attempted all the questions.

The information can be represented by the matrix $\mathbf{A} = \begin{pmatrix} 28 & 8 & 4 \\ 32 & 7 & 1 \\ x & 40-x & 0 \end{pmatrix}$.

Each correct answer will be awarded 2 marks.
For each incorrect answer, 1 mark will be deducted.
No marks will be awarded for questions that are not attempted.

(a) Find, in terms of x, the matrix $\mathbf{M} = \mathbf{A}\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$.

Answer $\mathbf{M} =$ [1]

(b) Explain what the matrix **M** represents.

.....

..... [1]

(c) The matrix **T**, where **T** = **BM**, represent the total marks Amos, Betty and Carol scored.
Write down the matrix **B**.

Answer $\mathbf{B} =$ [1]

(d) If Carol has scored the highest marks, using your answer in **(a)**, find her least number of correct answers.

Answer ..... [2]

12 Elly wants to find out how much time students spend on their mobile phone.
She uses this question on a questionnaire.

> How many hours do you spend on your mobile phone in a day?
> Tick one box.
>
> ☐ 1 to 4 ☐ 4 to 8 ☐ 8 or more

List **two** things wrong with this question.

1. .....

.....

2. .....

.....[2]

---

13 The line $5x + 6y = 31$ has the same gradient as the line $4x + ky = 11$.
Find the value of k.

Answer ..... [2]

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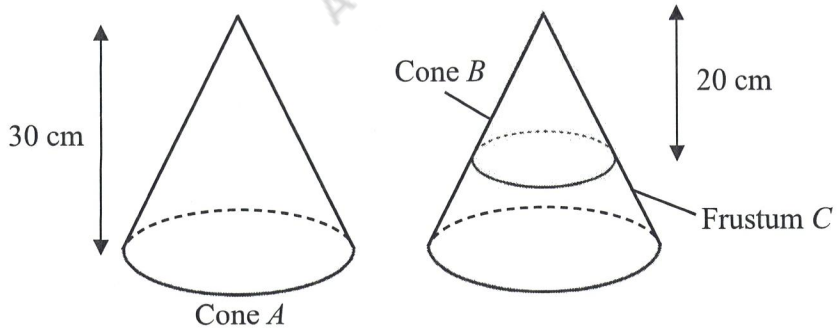
14 A smaller, geometrically similar cone B is removed from the top of an original cone A. The remaining solid is frustum C.



The heights of cone A and cone B are 30 cm and 20 cm respectively.

(a) Find the ratio of the diameter of the bases of cone A and cone B.

Answer ..... : ..... [1]

(b) Given that the curved surface area of cone A is 315 cm^2, find the curved surface area of the remaining solid frustum.

Answer ..... cm^2 [2]

(c) A similar third cone weighs 500g. Its base radius is $\frac{3}{5}$ of the base radius of cone A. Find the weight of cone A in kg.

Answer ..... kg [2]

15 **(a)** y is inversely proportional to the square of x.
Calculate the percentage change in y when x is halved.

Answer ..... % [3]

(b) A team of 15 men can paint a house in 8 days.
If 10 men left after 2 days, find the additional number of days the remaining men will need to work to complete painting the house, assuming all men work at the same rate.

Answer ..... [2]

16 The number of hours spent on social media by a group of 50 people in a company is recorded in the following frequency table.

Number of hours, x	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Frequency	2	3	6	14	18	7

(a) Calculate an estimate of the mean number of hours spent.

Answer ..... h [1]

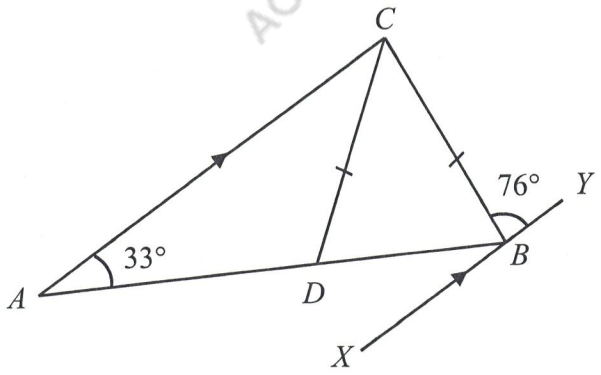
(b) Calculate an estimate of the standard deviation of the hours spent.

Answer ..... h [1]

17 Five positive integers have a mean of 10, a median of 9 and a mode of 6.
Given that the two highest integers are consecutive numbers, find the five numbers.

Answer .... [2]

18 In the diagram, AC is parallel to XY, $CD = CB$, angle $BAC = 33°$ and angle $CBY = 76°$.



(a) Find, with reasons,

(i) angle ABC,

Answer ..... [2]

(ii) angle ACD.

Answer ..... [1]

(b) A point M is such that DM is parallel to CB and BM is parallel to CD.
State the special name of the quadrilateral $BCDM$.

Answer ..... [1]

19 Triangle ABC is such that length $AB = 1$ cm, $BC = k$ cm and angle $ACB = \frac{\pi}{2}$.

(a) Explain why the value of k is less than 1.

.....

.....[1]

(b) Given $k = 0.56$, find angle BAC in radians.

Answer ..... [2]

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20 **(a)** Expand and simplify $(x-2y)(x+y)$.

Answer ..... [2]

(b) Simplify $\frac{3a^3b}{2a^2-7a+5} \div \frac{ab^3}{2a-5}$.

Answer ..... [2]

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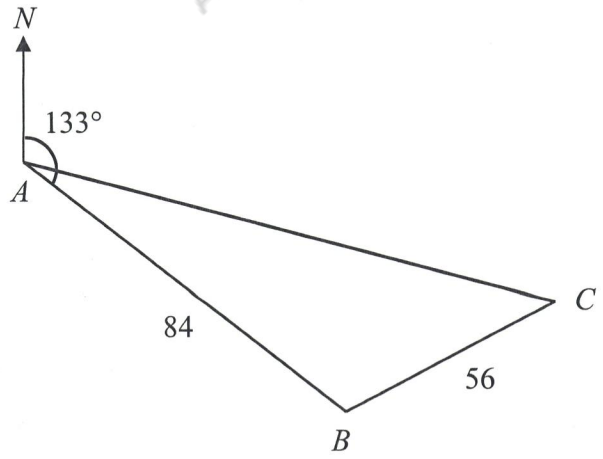
21 A car travels 84 km from A to B. The bearing of B from A is 133°.
It then travels to C which is 56 km away and on a bearing of 052° from B.



(a) Calculate

(i) the distance AC,

Answer ..... km [3]

(ii) the bearing of C from A.

Answer ..... [2]

(b) Show that the area of triangle ABC is 2323 km^2.

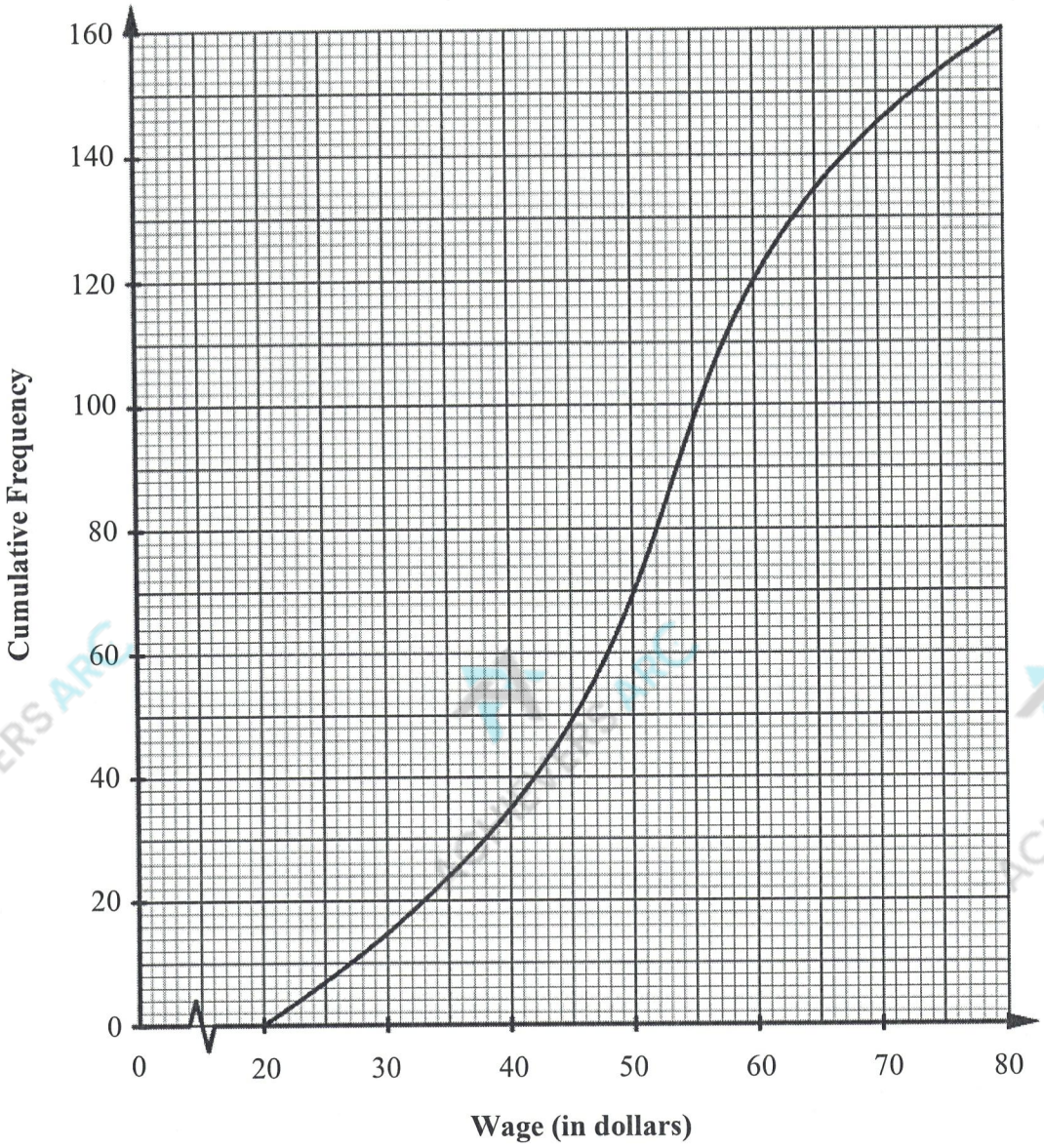
Answer

[2]

(c) Hence, find the shortest distance of B to AC.

Answer ..... km [2]

22 The cumulative frequency graph shows the daily wage, in dollars, of a random sample of 160 adults in Company *A*.



(a) Use the graph to estimate,

(i) the median wage,

Answer $..... [1]

(ii) the interquartile range of the wages.

Answer $..... [1]

(b) In this scenario, an 'outlier' is defined as any data value which is more than 1.5 times the interquartile range above the upper quartile.

Explain why none of the wages is high enough to be classified as an outlier.

.....

.....[2]

(c) The following cumulative frequency table shows the hourly wage for 300 employees in Company *B* and *C*.

Hourly wage (in dollars)	<10	<20	<35	<50	<70	<100
Cumulative Frequency (Company *B*)	25	68	159	234	260	300
Cumulative Frequency (Company *C*)	10	46	72	144	198	300

Without any calculations, explain why the median for Company *C* is higher than the median for Company *B*.

.....

.....

.....[1]

23 The position vectors of point A, B and C are $\begin{pmatrix} a \\ 2 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$ respectively, where a is a constant.

(a) Express $\overrightarrow{BC}$ as a column vector.

Answer ..... [1]

(b) Given that $\left|\overrightarrow{OA}\right| = \sqrt{5a}$, find all possible values of a.

Answer a = ... or ..... [2]

(c) Given that $\overrightarrow{OA} = k\overrightarrow{OB}$, explain why one of the values of a is not applicable.

.....

.....[1]

End of Paper

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2024
Secondary 4

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS

4052/02

Paper 2

Tuesday 20 Aug 2024

2 hours and 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Curved surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Solve the inequality $1 - 5x > -x + 3$.

Answer ..... [1]

(b) Solve $3 - 4x = \frac{6x - 1}{2}$.

Answer $x =$ ..... [1]

(c) It is given that $b = \frac{d + 3}{d} + c$.

(i) Find b when $c = -1$ and $d = 9$.

Answer $b =$ ..... [1]

(ii) Express d in terms of b and c.

Answer $d =$ ..... [3]

(d) Solve the equation $\frac{20}{x+2}+\frac{1}{6}=\frac{20}{x}$.

Give your solutions correct to two decimal places.

Answer x = ... or ..... [5]

2 **(a)** The planet Neptune is about 4.472 billion km away from the Sun.

(i) Write the distance between Neptune and the Sun in standard form.

Answer ..... km [1]

(ii) The planet Earth is about 1.496×10^6 km away from the Sun.
Given that the Sun, Earth and Neptune are aligned, find the shortest distance between Neptune and the Earth in standard form, correct to 4 significant figures.

Answer ..... km [1]

(iii) This table shows the planets and their corresponding sizes on a poster.

	Neptune	Earth
Actual Radius (km)	24622	r
Radius on Poster (cm)	9.85	2.55

Given that the scale used for the poster is the same for both planets, find the value of r.

Answer r = ... [2]

(b) Mr Tan bought a pen when Goods and Services Tax (GST) was 8% in 2023.
In 2024, when GST was increased to 9%, he bought the same item again.
If the cost of the pen, before GST, remains the same, calculate the percentage increase in the amount he paid for the pen.

Answer ..... % [3]

(c) The total cost of 2 televisions and 1 computer is $1470.
The total cost of 3 televisions and 2 computers is $2480.
Showing all necessary working, find the cost of a television.

Answer $..... [3]

3 P is the point (–3, 4) and Q is the point (21, 20).

(a) Find the length of the line PQ.

Answer ..... [2]

(b) Find the equation of the line PQ.

Answer ..... [2]

(c) The equation of line l is $3y - 2x = 11$.
Without solving, explain why line l does not intersect the line PQ.

Answer

.....

.....

..... [2]

(d) R and S are points that lie on the line l and M is a midpoint of PQ.
Given that the area of triangle PQR is 52.5 cm^2, find the area of triangle PMS.

Answer ..... cm^2 [1]

4 A goldsmith, Lim, has a solid right pyramid, made of gold, with a square base of side 10 cm and a slant height of 13 cm.

(a) Find the height of the pyramid.

Answer ..... cm [1]

(b) Find the volume of the pyramid.

Answer ..... cm^3 [2]

(c) Lim melts the gold and recasts it into a gold cylinder with a height of 15 cm.
Due to the melting and recasting process, the loss in the mass of gold ranges from 2% to 5%.
Find the maximum possible radius, r, of the cylinder.

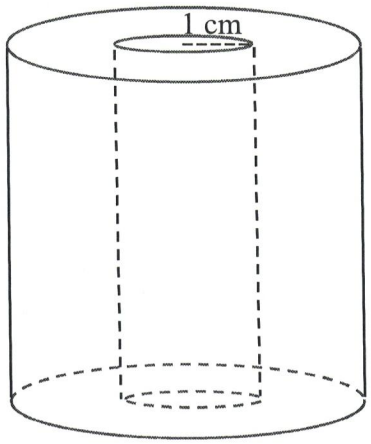
Answer r =..... cm [4]

(d) Given that the density of gold is 19.3 g/cm^3, show that the maximum mass of the gold cylinder is 7565.6 g.

Answer

[1]

(e) Lim wants to make identical gold rings using the gold cylinder of mass 7565.6 g. He drills a cylindrical hole of radius of 1 cm through the centre of the cylinder from top to bottom, as shown in the diagram below.



Find the number of identical rings, weighing 50 g each, Lim can make from the remaining solid.

Answer ..... [3]

(f) Another goldsmith, Tan, claims that from the same gold cylinder of mass 7565.6 g, he can make more gold rings, weighing 50 g each, than Lim.
Explain if Tan's claim is true.

Answer

.....

.....

..... [1]

5 **(a)** Complete the table of values for $y = -\frac{1}{2}x^3 + 3x - 2$.

x	–3	–2	–1	0	1	2	3
y	2.5	–4	–4.5	–2	0.5	0	

[1]

(b) On the grid opposite, draw the graph of $y = -\frac{1}{2}x^3 + 3x - 2$ for $-3 \le x \le 3$. [3]

(c) Explain how your graph shows that there are three solutions of the equation $-\frac{1}{2}x^3 + 3x - 2 = 0$.

.....

.....[1]

(d) Use your graph to find the positive value of k for which the equation $-\frac{1}{2}x^3 + 3x - 2 = k$ has exactly two solutions for $-3 \le x \le 3$.

Answer k = ... [1]

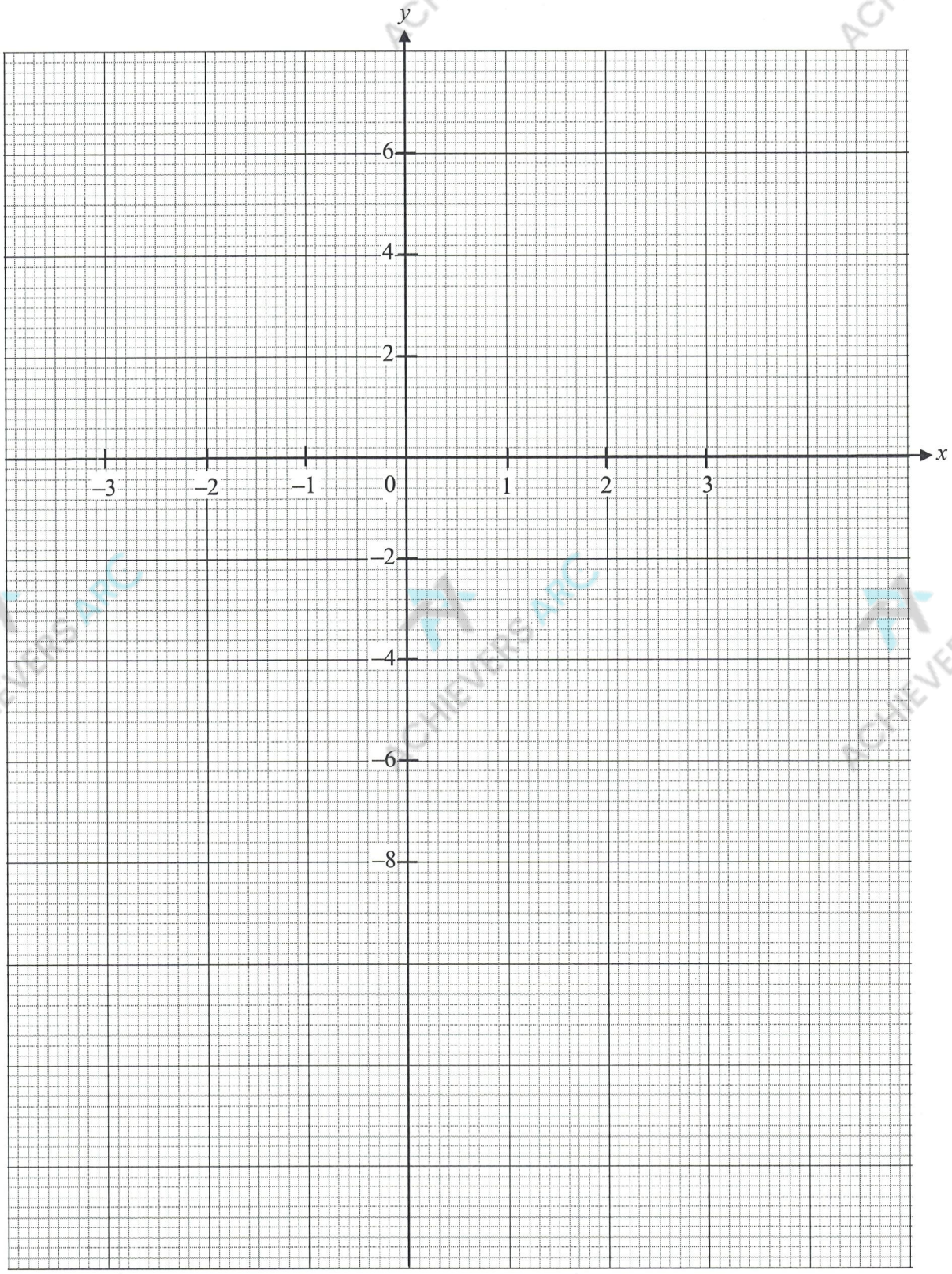
(e) **(i)** On the same grid, draw the line $y = -x - 4$ for $-3 \le x \le 3$. [1]

(ii) Write down the x-coordinates of the points where this line intersects the curve.

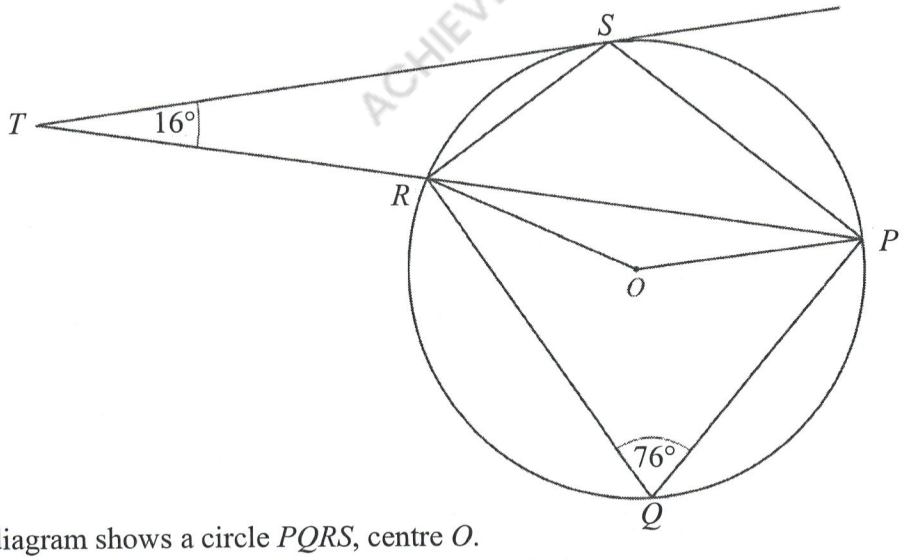
Answer x =..... or ..... [2]

(f) Another line $y = c - x$, where c is a constant and $c < 0$, intersects the curve at only one point. By drawing the line, use your graph to find the value of c.

Answer c = ... [2]


y
x
6
4
2
−2
−4
−6
−8
−3
−2
−1
0
1
2
3

6



The diagram shows a circle $PQRS$, centre O.
PRT is a straight line and ST is a tangent to the circle.
Angle $PTS = 16°$ and angle $PQR = 76°$.

(a) Giving reasons for each step of your working,

find

(i) angle PSR,

Answer Angle PSR =..... [1]

(ii) angle PRO.

Answer Angle PRO =..... [2]

(b) Show that triangle RST is similar to triangle SPT.

Give a reason for each statement you made.

.....

.....

.....

..... [2]

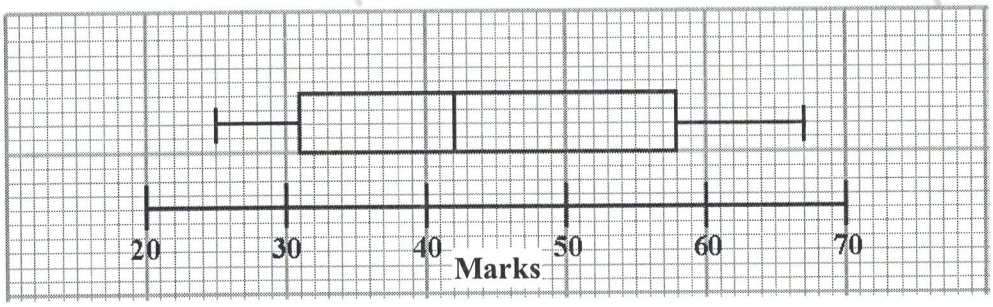
(c) Find angle *SPT*.

Answer Angle *SPT* =..... [2]

(d) Given that $3ST = 2PT$ and area of triangle *STR* is 28 cm^2, find the area of triangle *RSP*.

Answer ..... cm^2 [3]

7 **(a)** The distribution of marks obtained by 80 students for Mathematics Test 1 is summarised by a box-and-whisker plot shown below.



(i) Find

(a) the median mark,

Answer ..... [1]

(b) the interquartile range of the marks.

Answer ..... [2]

(ii) The same group of students sat for Mathematics Test 2.
The distribution of marks for Test 2 is summarised in the table below.

	Marks
25th percentile	30
Mean	48
Median	50
Upper Quartile	60

Make 2 comparisons between the marks for Test 1 and marks for Test 2.
Use figures to support your answers.

Answer

1. .....

.....

.....

2. .....

.....

.....

[3]

(b) The same group of 80 students sat for Additional Mathematics Test.

$\frac{1}{10}$ of them failed and a quarter of them scored distinction for the test.

(i) Two students are selected from the class at random.

(a) Find the probability that both students passed the test.

Answer ..... [2]

(b) Find the probability that one student failed the test and the other student passed the test but did not score a distinction.

Answer ..... [3]

(ii) A mistake has been discovered when recording the marks of the Additional Mathematics Test. To correct the mistake, 2 marks need to be added to all students' marks.
Describe the effect this correction would have on the mean and standard deviation of the marks of this test.

Answer

.....

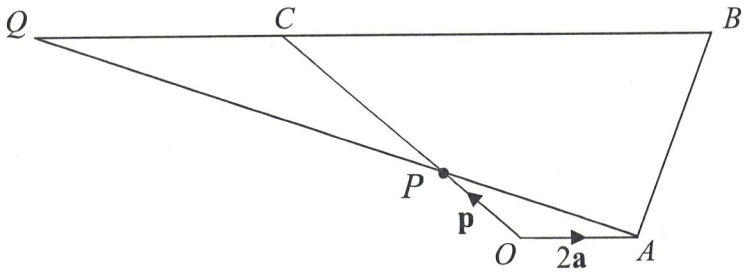
.....

.....

..... [2]

8 $OABC$ is a trapezium where OA is parallel to CB. $\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OP} = \mathbf{p}$.

$OA = \frac{1}{3}CB$ and Q is a point on AP produced such that $\overrightarrow{QB} = 9\mathbf{a}$.



(a) Find $\overrightarrow{QC}$.

Answer $\overrightarrow{QC}$ = ... [1]

(b) Find $\overrightarrow{PQ}$.
Give your answer as simply as possible in terms of **a** and **p**.

Answer $\overrightarrow{PQ}$ = ... [3]

(c) X is a point on AB such that PX is parallel to QB and that $\frac{\text{area of triangle } APX}{\text{area of triangle } AQB} = \frac{4}{25}$.

Find $\overrightarrow{AX}$, as simply as possible, in terms of **a** and **p**.

Answer $\overrightarrow{AX}$ = ... [3]

9 A small business, Mini Limited, uses the delivery service offered by Fast Delivery. The cost per use for the delivery services in 2024 are shown in the table below.

Vehicle Type	Base Fare	Distance Rate	Weight Limit
Motorcycle	$2	+ $0.35/km for the first 10 km + $0.40/km for the remaining distance	5 kg
MPV	$19	+ $0.50/km	50 kg
1.7m Van	$26	+ $0.50/km	400 kg
10-ft Lorry	$42	+ $0.50/km	1200 kg

Mini Limited has 2 business partners, JHL and DEX, situated 5 km and 15 km away from Mini Limited respectively.

(a) Show that the cost to send a document to JHL using the motorcycle is $3.75.

Answer

[1]

(b) Calculate the cheapest possible cost for Mini Limited to deliver 700 kg of goods to DEX.

Answer $..... [2]

(c) The table below shows the number of delivery services that Mini Limited has used from Fast Delivery in 2024.

	Delivery to JHL	**Delivery to DEX**
Motorcycle	17	16
1.7m Van	12	4

In addition, Mini Limited uses Fast Delivery to transport goods from a warehouse, which is 45 km away, to its office. As the weight of the goods is 1200 kg, a 10-ft lorry is needed for each of such delivery.

Mini Limited used this service **12** times in 2024.

Two other delivery companies, Swift Logistics and Speed Move, offer Mini Limited two annual packages for similar delivery services for the year 2025.

The packages are as follows:

	Speed's Package at $1280			**Swift's Package at $1450**		
Vehicle Type	**Usage Limit**	**Distance Limit**	**Weight Limit**	**Usage Limit**	**Distance Limit**	**Weight Limit**
Motorcycle	35	10 km	15 kg	33	18 km	5 kg
1.7m Van	20	10 km	400 kg	16	18 km	400 kg
10 ft Lorry	15	50 km	1200 kg	12	50 km	1200 kg

The services and charges will remain unchanged for Fast Delivery for the year 2025.

Which delivery service company should Mini Limited engage for the year 2025? Justify your decision and show your calculations clearly.

Answer

.....

..... [6]

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