TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2024
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS **4049/01**

Paper 1

Friday 16 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Find the set of values of k for which the line $y = 3x + k$ and the curve $y = 2x^2 - 3x + 4$ do not intersect. [4]

2 Solve $\operatorname{cosec}^2 x + \cot^2 x = \operatorname{cosec} x$ for $0° \le x \le 720°$. [5]

3 **(a)** Express $\dfrac{5x^2+2x+12}{(x^2+4)(x+1)}$ in partial fractions. [5]

(b) Hence, $\displaystyle\int \frac{5x^2+2x+12}{(x^2+4)(x+1)}\,\mathrm{d}x$. [3]

4 P is the point (8, 2) and Q is the point (12, 4).

(a) Point R lies on the x-axis such that R is equidistant from P and Q. Find the coordinates of R. [4]

(b) Point S is such that the four points, P, Q, R and S form a parallelogram where PQ and RS are parallel. Find the coordinates of the possible positions of S. [3]

5 (a) Prove the identity $\frac{\cos x}{1+\sin x} = \sec x - \tan x$. [3]

(b) Find $\frac{\mathrm{d}}{\mathrm{d}x}\ln(1+\sin 2x)$. [2]

(c) Using the results of **(a)** and **(b)**, find $\int \cos 3x + \sec 2x - \tan 2x \, \mathrm{d}x$. [4]

6 (a) A ball is thrown vertically upwards from a height of 4.5 m above ground. It falls on the ground 9 seconds after being thrown. The height, h m of the ball above the ground at time t seconds after being thrown is given by $h = at^2 + 4t + c$.

(i) Find the value of a and of c. [3]

(ii) Another ball is thrown vertically upwards from the same height and takes the same amount of time to hit the ground. Given that the second ball did not reach as high as the first ball, find the range of values of the maximum possible height that the second ball could have reached. [3]

(b) **(i)** Given that $x = -2(1-y)^4 + 9$ for all real values of y, explain why the maximum value of x is 9. [2]

(ii) By letting $b = (1-y)^2$, find the range of values of b for which $x < 5$. [3]

7 A dot moves on a screen and the movement can be modelled by the equation $y = \tan\left(2x + \frac{\pi}{4}\right)$, where x and y are the horizontal and vertical distances of the dot from the left and bottom of the screen respectively.

(a) Find the exact value of x at the first instant where y is moving at twice the rate at which x is moving. [5]

(b) The rate at which the x changes with respect to time, t, is given by $\frac{1}{6}e^{\frac{1}{2}t} - 3t$.

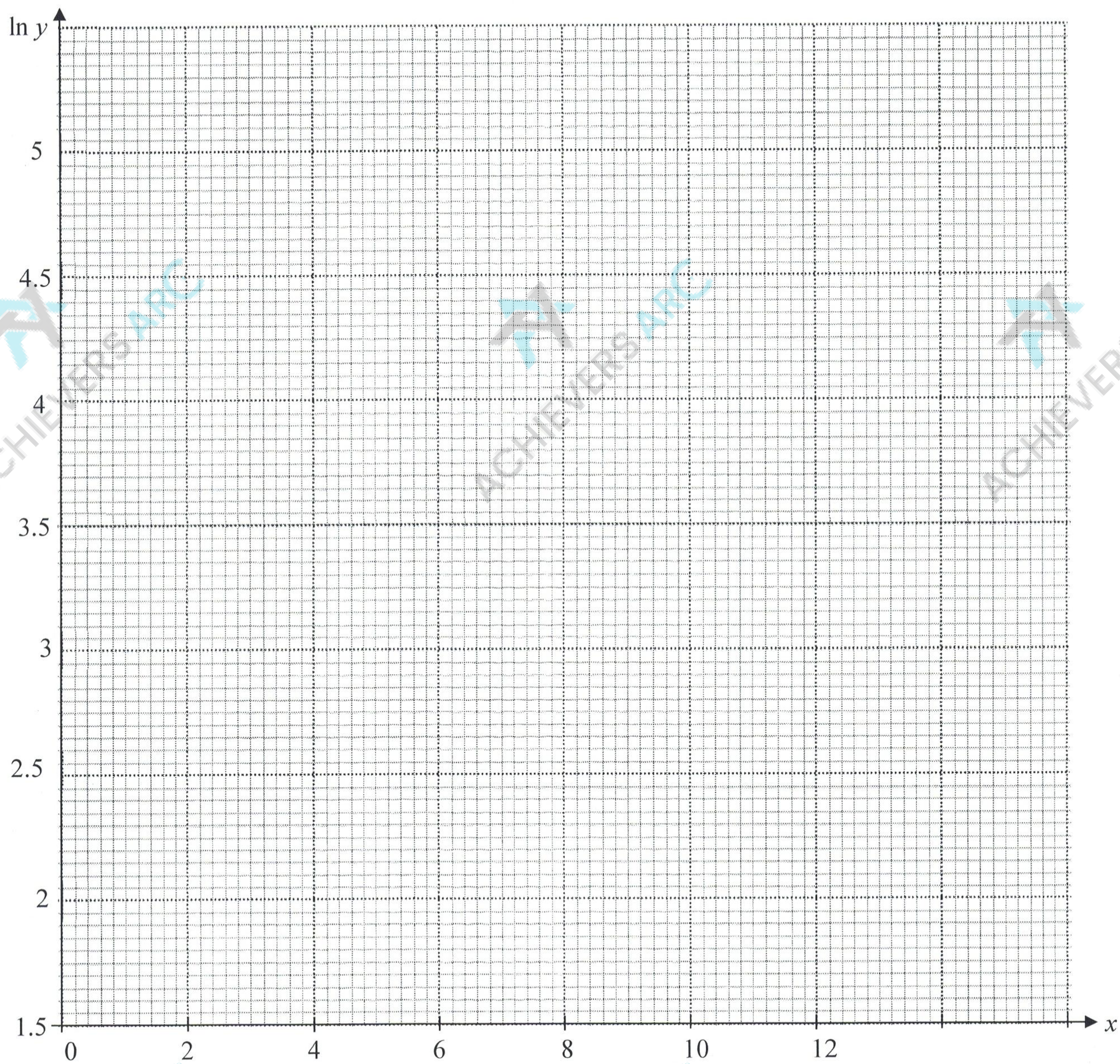
Given that $x = \frac{\sqrt{e}}{3}$ when $t = 1$, find an expression for x in terms of t. [4]

(c) Find the rate at which y changes at the start. [3]

8 The table shows some experimental values of the variables x and y which are related by the equation $y = An^x$, where A and n are constants.

x	2	4	6	8	10
y	9.8	19.4	37.4	74.0	144.4

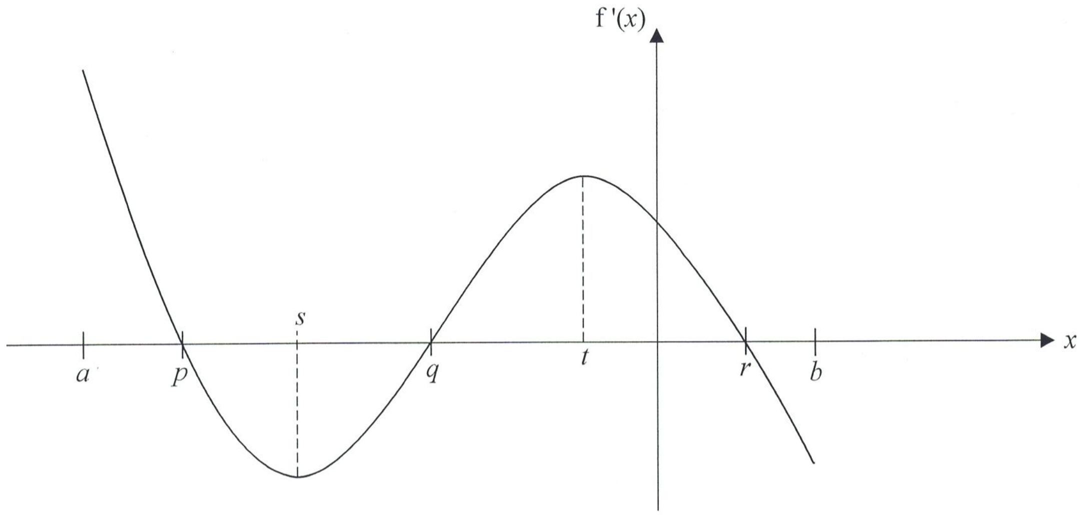
(a) Plot $\ln y$ against x and draw a straight line. [2]



(b) Use your graph to estimate the value of each of the constants A and n. [4]

(c) On the same diagram, draw the straight line representing $y = e^{-x+4}$ and hence find the value of x for which $An^x = e^{-x+4}$. [3]

9 Consider the function, f(x) where $a < x < b$. The following diagram shows the graph of f '(x), the derivative of f(x). The graph cuts the x-axis at $x = p$, $x = q$ and $x = r$ and has turning points at $x = s$ and $x = t$.



In terms of a, b, p, q, r, s and/or t, find

(a) the range of values of x when f(x) is increasing and justify your answer, [2]

(b) the value(s) of x when f(x) is maximum and justify your answer. [3]

10 The equation of a curve is $y = \dfrac{3x^2}{x+1}$, where $x \neq -1$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x^2 + 6x}{(x+1)^2}$ and find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [6]

(b) Find the coordinates of the stationary points on the curve. [3]

11 The centre of a circle, C is $(5, -3)$. The equation of the tangent to the circle at a point A, is $y = \frac{4}{3}x - \frac{4}{3}$.

(a) Show that the x-coordinate of A is 1. [4]

(b) Find the equation of the circle. [3]

(c) The equation of another circle is given by $(x+3)^2+y^2+14y+40=0$.
Determine, with reasons, whether the two circles will overlap. [4]

End of Paper

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2024
Secondary 4

CANDIDATE NAME

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ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **Wednesday 21 August 2024**

2 hours 15 minutes

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1 Find the value of q given that the coefficient of x in the expansion of $(2+3x)^4\left(1-\dfrac{x}{8}\right)^8-(1-qx)^2$ is zero. [5]

2 It is given that $9^{x-1} \times 2^{2x+2} = 6^{3x}$.

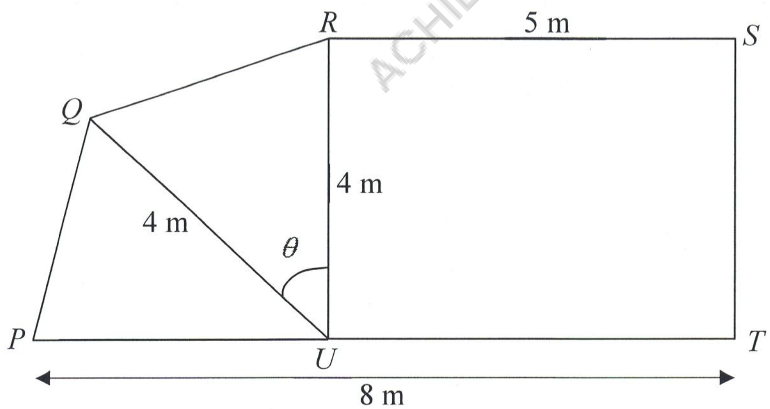
(a) Find the exact value of 6^x. [3]

(b) Solve the equation $9^{x-1} \times 2^{2x+2} = 6^{3x}$. [2]

3 The population, P, of a certain bacteria can be modelled by the equation $P = Ae^{nt}$, where A and n are constants and t is the time in hours.
There were approximately 1000 bacteria at the start and the population, P, doubles every 6.5 hours.

Find the population of bacteria after a day and a half, leaving your answer correct to 4 significant figures. [4]

4



In the diagram, $PQRST$ is a garden where $RSTU$ is a rectangle and PUT is a straight line.
It is given that $RS = 5\,\text{m}$, $PT = 8\,\text{m}$, $QU = RU = 4\,\text{m}$ and $\angle QUR = \theta$.
The area of the garden $PQRST$ is A m^2.

(a) Show that A can be expressed as $20 + 8\sin\theta + 6\cos\theta$. [3]

(b) Express A in the form $a + R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

(c) Given that θ can vary and the owner paid \$1200 for fertilising the whole garden, find the minimum cost per m^2 of fertilisers and the corresponding value of θ. [3]

(d) Explain whether the area of the garden $PQRST$, A, can be equal to $20\ m^2$. [2]

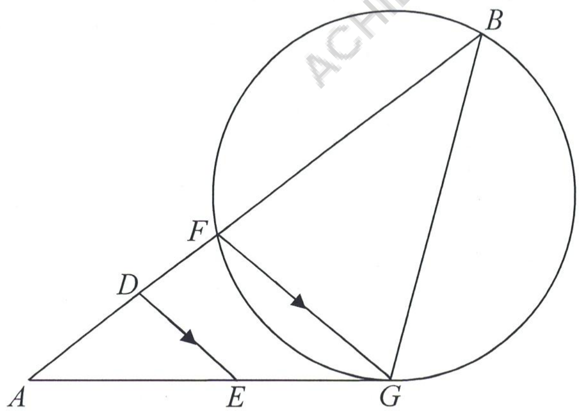
5 (a) By expressing $\sin 3x = \sin(2x + x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$. [3]

(b) Solve the equation $6\sin x = 1 + 8\sin^3 x$ for $-\frac{2\pi}{3} < x < \frac{2\pi}{3}$.

Leave your answers in exact value. [4]

(c) State the maximum value of $5 + 16\sin^3 x - 12\sin x$. [2]

6



The diagram shows 3 points B, F and G on a circle. The 3 points also lie on triangle ABG. It is given that DE is parallel to FG.
E is the midpoint of AG and AG is a tangent to the circle at G.

(a) Show that triangle AFG is similar to triangle AGB. [2]

(b) Prove that $AB \times AF = 4AE^2$. [2]

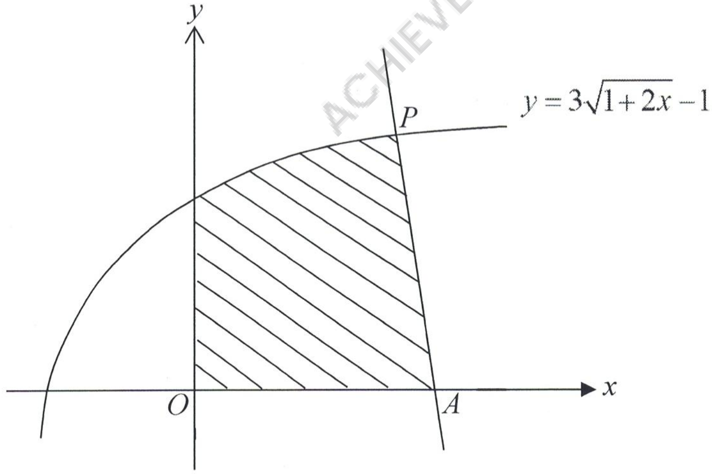
7 A moving particle travelling in a straight line passes a fixed point O with a velocity of $10\ \text{ms}^{-1}$. Its acceleration, $a\ \text{ms}^{-2}$, is given by $a = 3 - 4\sin t$, where t is the time in seconds after passing O.

(a) Calculate the displacement of the particle at $t = 2$. [6]

(b) Find the velocity at that instant. [2]

(c) Justify why the particle will never return to O. [2]

8



The diagram shows part of the curve $y = 3\sqrt{1+2x} - 1$. The normal to the curve at the point $P(a,\ b)$ cuts the x-axis at A.

Given that the gradient of this normal is -2, calculate

(a) the value of a and of b, [5]

(b) the area of the shaded region. [6]

9 It is given that $\cos 2\theta = x + y$ and $\sin 2\theta = x - y$.

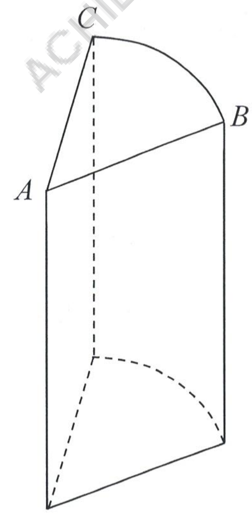
(a) Show that $2\left(x^2 - y^2\right) = \sin 4\theta$. [2]

(b) By considering $(x+y)^2$ and $(x-y)^2$, evaluate the value of $x^2 + y^2$. [4]

(c) Given further that $\frac{x}{y} = -\frac{1}{7}$, deduce that the value of $\tan 2\theta = -\frac{4}{3}$. [3]

(d) Without using a calculator, find the possible values of $\tan\theta$. [3]

10



The diagram shows a solid prism where ABC is a sector of a circle, centre A.
The arc length $BC = r$ m, where r is the radius of the circle.
It is given that the volume of the prism, V m^3, is a constant.

[Arc Length $= r\theta$, Sector Area $= \frac{1}{2}r^2\theta$, where θ is in radians]

(a) Show that the total surface area of the prism, A m^2, is given by $A = r^2 + \frac{6V}{r}$. [4]

(b) Given that r can vary, find an expression for r, in terms of V, for which A has a stationary value. [3]

(c) It is given that $V = 9\,\text{m}^3$. Find the value of r which gives a minimum value of A. [2]

(d) Determine whether there is a value of r such that the prism has a surface area of 25 m^2, where $V = 9$ m^3. [2]

11 **(a)** Showing your working clearly, solve the equation

$$2\left(27^x\right)+9\left(9^x\right)=6-7\left(3^x\right).$$ [6]

(b) Hence solve the equation $2\left(\frac{3^x}{9}\right)^3+9\left(\frac{3^x}{9}\right)^2+7\left(\frac{3^x}{9}\right)=6$. [2]

End of Paper