3

Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

19 August 2024 Monday 2 hours 15 min

2024 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use													
Qn	1	2	3	4	5	6	7	8	9	10	11	12	*Marks Deducted*
Marks													

Category	Accuracy	Units	Notations	Others
Question No.				

TOTAL MARKS
90

Setter: Ms Sabrina Tan
Vetter: Mr Tan Lip Sing

This question paper consists of **19** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Express $\dfrac{3x^2 + 14x + 6}{x^2(x + 3)}$ as the sum of 3 partial fractions. [5]

2 **(a)** State, in terms of π,

(i) the principal value of $\tan^{-1}(-\sqrt{3})$, [1]

(ii) the values between which the principal value of $\sin^{-1}x$ must lie. [1]

(b) Given that A is a reflex angle and $\cos A = -\dfrac{4}{5}$, find the exact value of $\cos(A + 30°)$ without the use of a calculator. [3]

3 **(a)** Find the set of values of the constant k for which the curve $y = -kx^2 - 2x + 2k - 3$ does not intersect the x-axis. [3]

(b) Using the answer in **part (a)**, explain whether it is possible for $-kx^2 - 2x + 2k - 3$ to be positive for all x. [2]

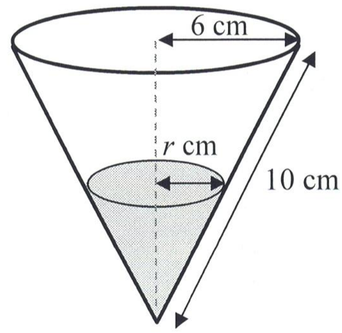
4 A roller coaster is being designed such that the height, h m, of a rider above the ground in a section of the roller coaster ride is given by $h = 10x - 2x^2 - 4$, where x is the horizontal distance of the rider from the starting point and $1 \leq x \leq 4$.

(a) Express h in the form $a + b(x + c)^2$ where a, b and c are constants to be determined. [3]

(b) Hence, explain why the rider cannot reach a height of 10 m. [1]

(c) After testing the prototype, the roller coaster designer wants to make the ride more exciting by moving the highest point of this section of the roller coaster ride up by 1.2 m and left by 0.1 m. Write down a possible new expression for h. [1]

5 Water is poured into an empty inverted conical container with radius 6 cm and slant height 10 cm. After t seconds, the radius of the top surface of the water is r cm.



(a) Show that the surface area, S, of water in contact with the container at any time is given by $S = \frac{5}{3}\pi r^2$.

[The curved surface area of a cone of base radius r and slant height l is πrl.] [2]

(b) Water is poured into the container such that r increases at a constant rate. Given that it takes 30 seconds to completely fill up the empty container, write down the rate at which r increases. [1]

(c) Hence calculate the rate at which S increases when the container is one-eighth filled. [4]

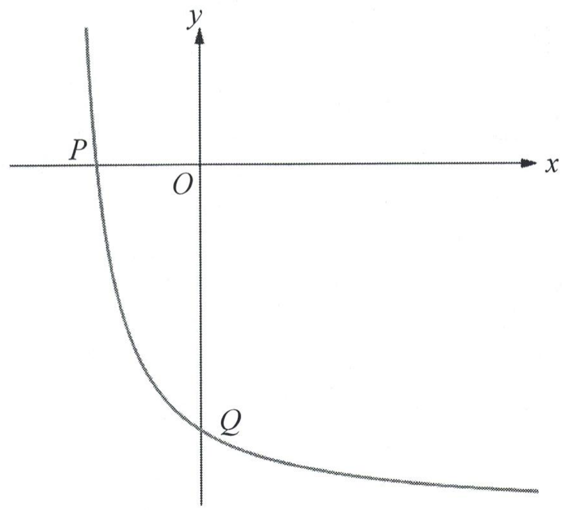
6 A curve has equation $y = \dfrac{30(2x-1)^3}{e^{3x}}$, where $x > 0$.

(a) Find the x-coordinates of the stationary points of the curve. [5]

(b) Determine the nature of each of the stationary points. [3]

(c) Using your answer in **part (b)**, infer and write down the set of values of x for which y is an increasing function. [2]

7 The diagram shows part of the curve $y = \frac{40}{x+8} - 20$, where $x > -8$. The curve intersects the x-axis and y-axis at P and Q respectively. The tangent to the curve at R is parallel to line PQ.

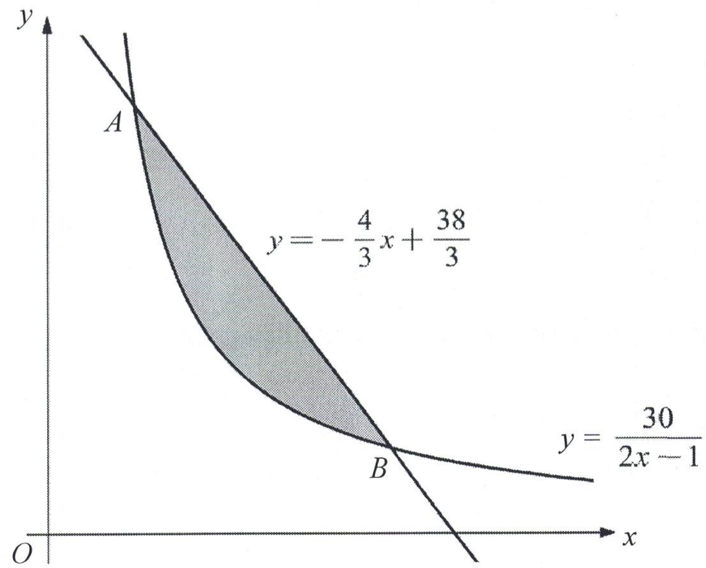


(a) Show that the coordinates of point R are $(-4, -10)$. [5]

(b) Given that the point S has coordinates $(-2, -5)$, find the area of the quadrilateral $PRQS$. [2]

(c) Determine, with reason, whether $PRQS$ is a parallelogram. [3]

8 The diagram shows part of the curve $y = \dfrac{30}{2x-1}$ and the line $y = -\dfrac{4}{3}x + \dfrac{38}{3}$. The curve intersects the line at the points A and B.



(a) Find the coordinates of the points A and B. [3]

(b) Find the area of the shaded region. [4]

(c) Show that the area bounded by the curve, the y-axis and the lines $y = 2$ and $y = 6$ can be expressed in the form $15\ln p + q$, where p and q are constants to be determined. [3]

9 The function f is given by $f(x) = x^3 - x^2 + ax - 1$ for $x \neq 1$.

It is given that , where $Q(x)$ is a quadratic function.

(a) Explain how you can use Remainder Theorem to show that $a = 4$. [3]

(b) By using long division, divide $f(x)$ by $x^2 + 1$.

Hence, express $\frac{f(x)}{x^2+1}$ in the form $px + q + \frac{rx}{x^2+1}$, where p, q and r are constants to be determined. [2]

(c) Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln(x^2+1)$. [1]

(d) Hence, using your results from **part (b) and (c)**, find $\displaystyle\int \frac{\mathrm{f}(x)}{x^2+1}\,\mathrm{d}x$. [2]

10 The equation of a circle, C_1, with centre A, is $x^2 + y^2 - 6x + 8y - 75 = 0$.

(a) Find the radius of the circle and the coordinates of its centre, A. [4]

(b) A second circle, C_2, with centre B, has radius 12 units.

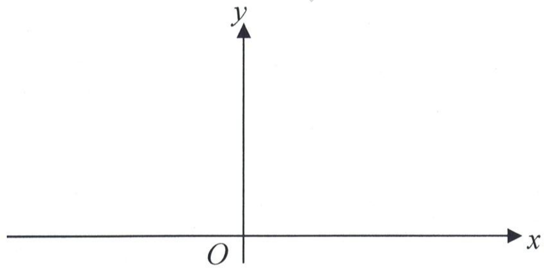
The equation of the perpendicular bisector of AB is .

Find the equation of the second circle, C_2. [5]

(c) P is a point on the perpendicular bisector of AB, where P is **not** the midpoint of A and B.

Explain whether triangle PAB is an isosceles triangle. [1]

11 **(a)** Sketch the graph of $y = e^{-x}$ on the given axes. Label any axial intercepts. [1]

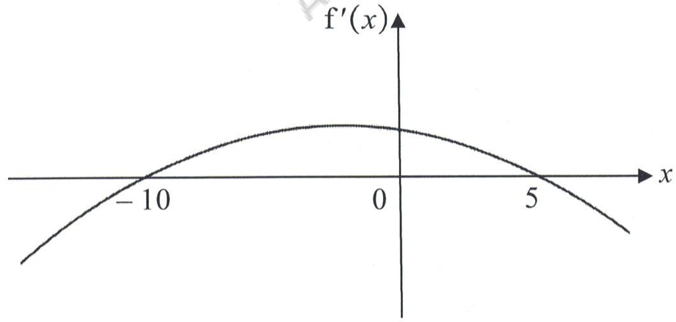


(b) The curve $y = f(x)$ is such that $f'(x) = e^{2x+1} + e^{-x}$.

(i) Explain why the curve has no stationary points. [2]

(ii) The curve passes through the point (ln 4, 8e). Without the use of a calculator, find an expression for $f(x)$. Show all working clearly. [6]

12 The diagram shows the graph of $y = \mathbf{f}'(x)$.



(a) Write down the range of values of x for which the **function f** decreases as x increases. [1]

(b) The above function f is given by $f(x) = 12k^2x - 2x^3 - 3kx^2$, where k is a positive constant. By solving a suitable inequality, use the answer in **part (a)** to find the value of the constant k. [4]

(c) Explain how you can use the above graph of $y = f'(x)$ to determine the value of x for which f(x) is a maximum value. [1]

END OF PAPER

Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

20 August 2024 Tuesday 2 hours 15 min

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Marks											

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Question No.				

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Setter: Mr Gregory Quek
Vetter: Mr Tan Lip Sing

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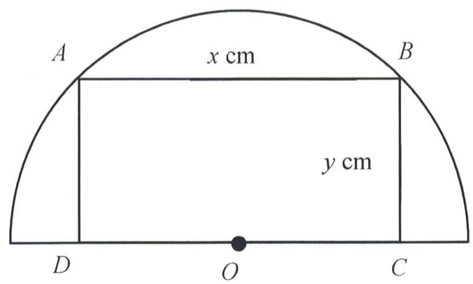
1 **(a)** Write down, and simplify, the first three terms in the expansion of $\left(3-\frac{2}{x}\right)^5$ in descending powers of x. [2]

(b) Given that there is no term independent of x in the expansion of $\left(5+ax^2\right)\left(3-\frac{2}{x}\right)^5$, hence find the value of the constant a. [3]

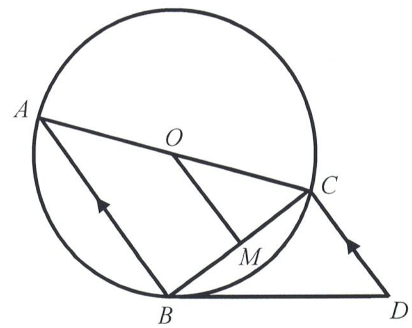
2 In the figure, $ABCD$ is a rectangle inscribed within a semicircle of radius 4 cm and centre O. It is given that $AB = x$ cm and $BC = y$ cm.



(a) Show that the area of the rectangle, A cm, is given by $A = \frac{1}{2}x\sqrt{64 - x^2}$. [2]

(b) Find the exact value of x for which A has a stationary value.
Give your answer in the form $k\sqrt{2}$, where k is an integer. [4]

3 The diagram shows a triangle ABC inscribed in the circle with centre O.
BD is a tangent to the circle at B and AB is parallel to CD. Point M is the midpoint of BC.



(a) Prove that triangles ABC and BCD are similar. [3]

(b) Prove that $ABMO$ is a trapezium. [2]

(c) Prove that $OM = \dfrac{BC^2}{2CD}$. [3]

4 Milk is poured into an empty cup and heated. The temperature, T_m °C, of the milk in the cup, t minutes after it is heated, is modelled by the formula, $T_m = 5(2)^t + 20$.

(a) State the initial temperature of the milk. [1]

Coffee is poured into another empty cup. The temperature, T_c °C, of the coffee in the cup, t minutes after it is poured, is modelled by the formula, $T_c = 60(2)^{-t} + 25$.

(b) Find the time taken for the temperature of the coffee to drop to 35°C. [3]

(c) Find the time taken for the milk and the coffee to reach the same temperature. [4]

5 It is given that $f(x)=2x^3-x^2y-13xy^2-6y^3$.

(a) Show that $x-3y$ is a factor of $f(x)$. [2]

(b) If $y=1$, find an expression in fully factorised form for $f(x)$. [3]

(c) Hence solve the equation $2e^{6z} - e^{4z} - 13e^{2z} - 6 = 0$ and show that the solution may be written in the form $\ln\sqrt{p}$, where p is an integer. [3]

6 **(a)** Given that $\tan\theta = 2\operatorname{cosec}\theta$, show that $\cos^2\theta + 2\cos\theta - 1 = 0$. [3]

(b) Using **part (a)**, find the exact value of $\cos\theta$ in simplest form, given that [3]

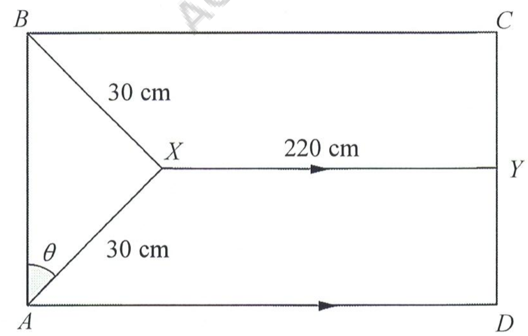
(c) Hence find the value of $\sec^2\theta$ in the form $a+b\sqrt{2}$, where a and b are integers. [5]

7 **(a)** Prove that $(\sin 2x)(\cot x) - 1 = \cos 2x$. [2]

(b) Given that $y = (\sin 2x)(\cot x) - 1$, hence show that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 3\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) + 2y + 9\sin 2x = 0$ may be written in the form $\tan 2x = k$, where k is a constant to be found. [4]

(c) Solve $\tan 2x = -\sqrt{3}$ for $0 \leq x \leq 2\pi$, giving your answers in terms of π. [4]

8



The diagram shows a rectangular flag $ABCD$. XAB is a triangle with $AX = BX = 30$ cm and angle $XAB = \theta$ for $0 < \theta < 90°$. XY is parallel to AD and $XY = 220$ cm.

(a) Express the area of triangle XAB in the form $q\sin 2\theta$, where q is an integer. [2]

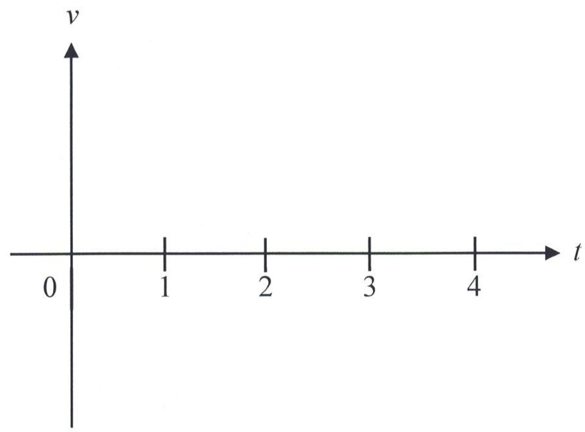
(b) Given that θ can vary, find the maximum possible area of triangle XAB and the value of θ at which this occurs. [2]

(c) Show that the perimeter, P cm, of the rectangular flag $ABCD$ can be expressed in the form $a\sin\theta + b\cos\theta + c$, where a, b and c are constants to be found. [3]

(d) By expressing P in the form $R\sin(\theta + \alpha) + c$, where $R > 0$ and $0 < \alpha < 90°$, explain if it is possible to have a flag with perimeter 550 cm. Show your working clearly. [5]

9 A particle moves in a straight line so that, t seconds after passing a fixed point O, its velocity, v metres per second, is given by $v = \pi\cos(\pi t) + \pi$.

(a) Sketch the velocity-time graph of the particle for $0 \le t \le 4$. [3]



(b) Determine how many times the particle is at instantaneous rest in the first 10 seconds. [1]

(c) Explain why the particle will never return to the origin O. [2]

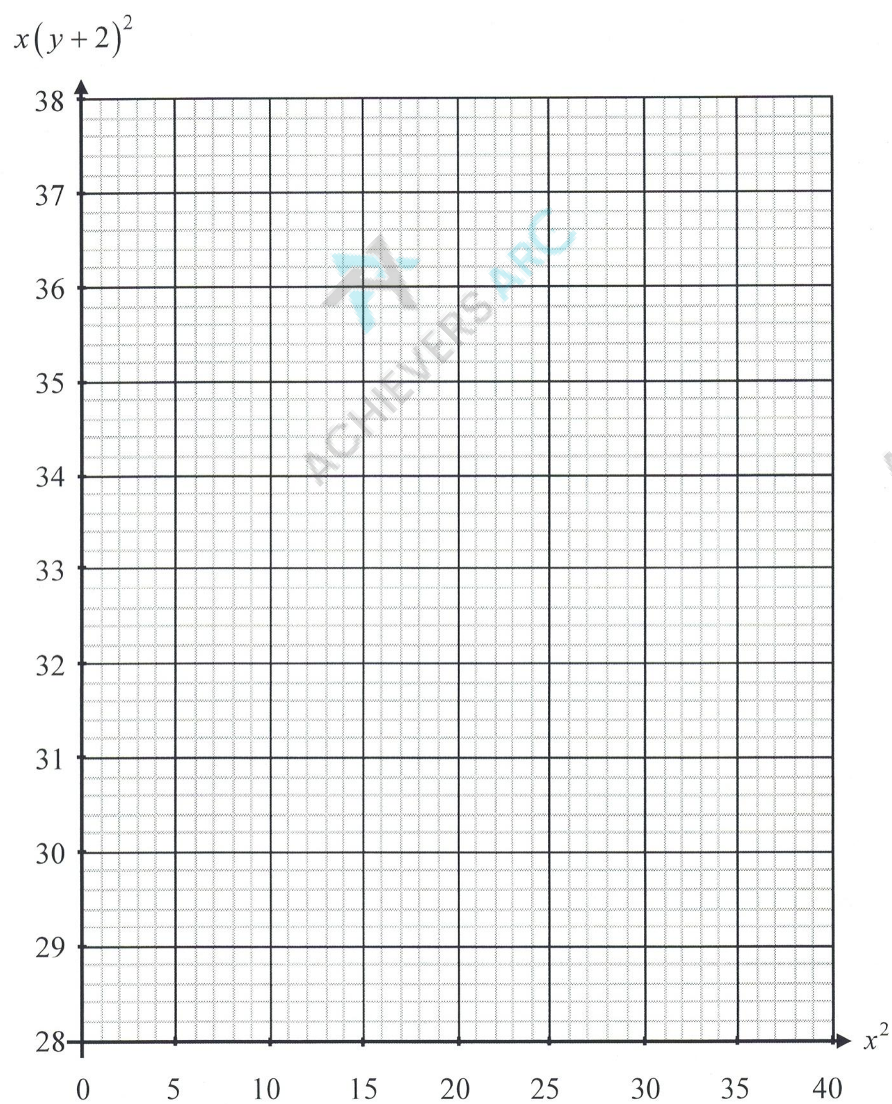
(d) Find an expression, in terms of t, for the displacement of the particle. [2]

(e) Calculate the average speed of the particle in the first 4 seconds. [3]

10 It is known that x and y are related by the equation $y = \sqrt{Ax + \frac{B}{x}} - 2$, where A and B are positive constants. The following table shows the values of the variables, x and y.

x	2	3	4	5	6
y	1.92	1.26	0.881	0.646	0.490

(a) Plot $x(y+2)^2$ against x^2 and draw a straight line graph to illustrate the information. [3]



(b) Express the equation $y=\sqrt{Ax+\frac{B}{x}}-2$ in a form that will yield the straight line graph in **part (a)**. [2]

(c) Use your graph to estimate the value of A and of B. [2]

(d) Explain why the graph $y=\sqrt{Ax+\frac{B}{x}}-2$ is undefined for $x \le 0$. [2]

(e) By drawing a suitable line on your graph, estimate the value of x for which $y+2=\frac{6}{\sqrt{x}}$.

Give your answer to 3 significant figures. [2]

END OF PAPER