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中正中学 文顺

CHUNG CHENG HIGH SCHOOL (YISHUN)

2024 Preliminary Examination
Secondary Four Express / Five Normal Academic

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS
Paper 1

4049/01
22 August 2024
2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need of clear presentation in your answers. Up to 2 marks may be deducted for improper presentation.

The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use		
Presentation Deduction		–1 / –2
TOTAL	90	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGNOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 **(i)** Given that the line $y = 2x - k$ meets the curve $y = \frac{k}{4}x^2 - 2kx + 1$, find the range of values of k. [4]

(ii) A student claimed that when $k = -\frac{1}{2}$, the curve does not intersect the line. By showing your workings clearly, explain whether the statement is valid. [2]

2 A triangle has a base of $\left(\frac{1}{\sqrt{2}}-\frac{\sqrt{32}}{2}+\frac{16}{\sqrt{48}}\right)$ m and a height of h m. Given that the area of the triangle is $\left(2\sqrt{2}-\sqrt{3}\right)$ m^2, find, without using calculator, the value of h in the form $\frac{a\sqrt{6}+b}{5}$ where a and b are integers. [5]

3 **(i)** Show that $3x^2 - 5x + 21$ is always positive for all real values of x. [3]

(ii) The curve $y = ax^n$, where a and n are constants, passes through $(2, 48)$, $(3, 108)$ and $(k, 192)$. Find the values of a, n and k where $k > 0$. [3]

4 **(a)** Given that $f(x)=\dfrac{2x+3}{\sqrt{4x-3}}$, find $f'(x)$. [3]

(b) Find the set of values of x for $y=\ln\left(\dfrac{3+8x}{8x-5}\right)$ to be a decreasing function. [3]

5 **(a)** Explain why $\dfrac{2x^3-4x^2+13x-10}{(x^2+4)(x-2)}$ is considered an improper fraction. [1]

(b) Express $\dfrac{2x^3-4x^2+13x-10}{(x^2+4)(x-2)}$ in the form of $A+\dfrac{Bx+C}{(x^2+4)(x-2)}$ and hence, express $\dfrac{2x^3-4x^2+13x-10}{(x^2+4)(x-2)}$ in partial fractions. [6]

6 **(a)** Given that $6x^2-9x+18=A(2x+1)(x-1)+B(x-2)+C$ for all values of x, find the values of A, B and C. [3]

(b) Given that $f(x)=2x^3-11x^2+3x+36$, show that $2x+3$ is a factor of $f(x)$ and hence solve the equation $f(x)=0$. [5]

7 Without using a calculator, evaluate $\dfrac{\log_5 9 + 2\log_5 6 - 4\log_5 3}{\log_{25} 4}$. [4]

8 Simplify $2^{3x-4} \times 9^{x-3} = 6^{2(x-2)}$ in the form $\left(\dfrac{m \times n}{36}\right)^x = 9$, where m and n are integers.

Hence solve for x, giving your answer in the simplest form $\dfrac{\ln a}{\ln b}$, where a and b are integers.

[5]

9 The function f is defined, for $0° \le x \le 180°$, by $f(x) = c + a\cos bx$, where a, b and c are integers. Given that the amplitude of f is 4 and the period of f is $120°$,

(i) state the value of a and of b, [2]

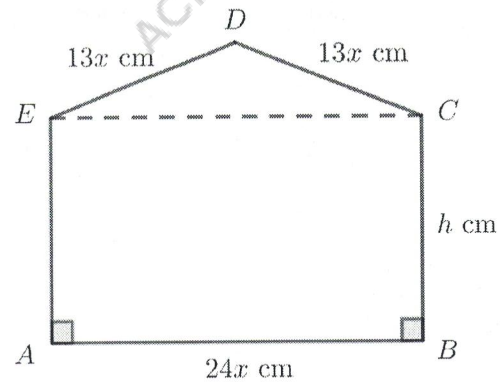
Given that the minimum value of f is -5,

(ii) find the value of c, [1]

(iii) sketch the graph of $y = f(x)$ for $0° \le x \le 180°$. [3]

(iv) A line $y = k$ is drawn on the graph of $y = f(x)$. State the range of values of k that will give 3 intersections. [1]

10 The diagram shows an outline of a board, $ABCDE$, consisting of a rectangle $ABCE$ of height h cm and width $24x$ cm and an isosceles triangle CDE in which $CD = DE = 13x$ cm. The perimeter of the board is 480 cm.



(a) Show that the area of the board, A cm^2, is given by $A = 5760x - 540x^2$. [3]

(b) Calculate the stationary value of A and determine whether it is a maximum or a minimum. [5]

11 **(a)** If $a\cos^2\theta + b\sin^2\theta = c$, show that $\tan^2\theta = \dfrac{a-c}{c-b}$. [4]

(b) **(i)** Given that $\sqrt{(2\cos^2 x - \sin x)} = 2\sin x$, show that $6\sin^2 x + \sin x - 2 = 0$. [2]

(ii) Hence, solve the equation $\sqrt{(2\cos^2 x - \sin x)} = 2\sin x$ for $0 \le \theta \le 2\pi$. [4]

12 The positive x- and y-axes are tangents to a circle C with equation $(x-a)^2+(y-b)^2=r^2$.

(a) What condition(s) must apply to the constants a, b and r? [1]

(b) It is given that $N(1, 2)$ is a point on the circle and it is further from the centre of the circle C as compared to the origin. Show that the coordinates of the centre M are $(5,\ 5)$. [4]

(c) Find the equation of the tangent to C at the point N. [3]

(d) The tangent intersects the axes at the points P and Q. The point R has coordinates $\left(k^2, -k\right)$, where $a > 2$. Find the area of the quadrilateral $MPQR$ in terms of k. [4]

13 **(a)** Show that $\sin 2x \cot x = 2\cos^2 x$. [1]

(b) Hence, without using a calculator, find the value of the constant k for which

$$\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \operatorname{cosec} 6x \tan 3x \, dx = \frac{\sqrt{3}}{k}.$$ [5]

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4049/02

Paper 2

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1 (a) Find, in terms of n, the coefficient of x^4 in the expansion of $\left(2-5x^2\right)^n$.
Write your answer in its simplest form. [2]

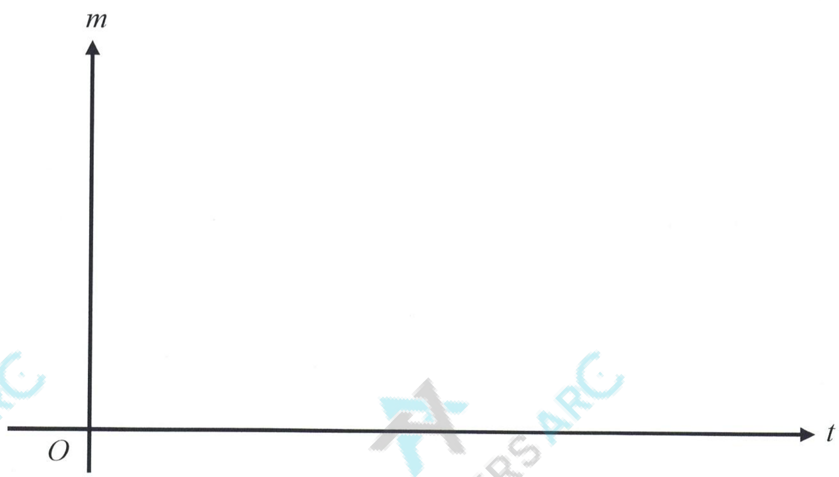
(b) (i) Write down the first three terms, in ascending powers x, in the binomial expansion of $\left(1+ax\right)^8$, where a is a constant. [2]

(ii) Given that the expansion of $\left(1-3x\right)^2\left(1+ax\right)^8$ in ascending powers of x is $1+10x+bx^2+\ldots$, calculate the value of a and of b. [3]

2 The mass, m grams, of a radioactive substance remaining, t days after being measured is given by $m = 12e^{-0.01t} + 0.5$.

(a) Find the initial mass. [1]

(b) Sketch the graph of $m = 12e^{-0.01t} + 0.5$ for $t \geq 0$. [2]



(c) Find the least number of complete days needed before the amount of substance is reduced to less than 5% of its initial mass. [3]

(d) Explain why the mass of the radioactive substance can never be less than 0.5 g. [2]

3 **Do not use a calculator for this whole question.**

(a) It is given that $\tan A = \frac{8}{15}$ and $\sin B = -\frac{3}{5}$, and that A and B are in the same quadrant.

Calculate the values of

(i) $\sin 2B$, [2]

(ii) $\cos(A+B)$. [2]

(b) Use the identity for $\tan 2A$ to show that $\tan 75° = 2 + \sqrt{3}$. [5]

4 A curve has equation $y = 4x\sin 2x$.

If $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 4y = A\sin 2x + B(x+2)\cos 2x$, find the value of A and of B. [5]

5 **(a)** The curve $y = f(x)$ is such that $f'(x) = 3e^{x} + e^{-2x}$.

(i) Explain why the curve $y = f(x)$ has no stationary points. [2]

(ii) Given that the curve passes through the point $(0, 2)$, find the expression for $f(x)$. [2]

(b) **(i)** Given that $y = (4x - 5)e^{-3x}$, find $\frac{dy}{dx}$. [2]

(ii) **Hence** evaluate $\int_0^2 4xe^{-3x}\,dx$, giving your answer correct to 3 significant figures. [3]

6 A particle moves in a straight line such that, t seconds after leaving a fixed point O, its velocity, v m/s, is given by $v = t^2 - 9t + 8$. The particle comes to instantaneous rest firstly at A and then at B.

(a) Find an expression, in terms of t, for the displacement of the particle from O. [2]

(b) Find the total distance travelled by the particle in the first 5 seconds after passing O. [4]

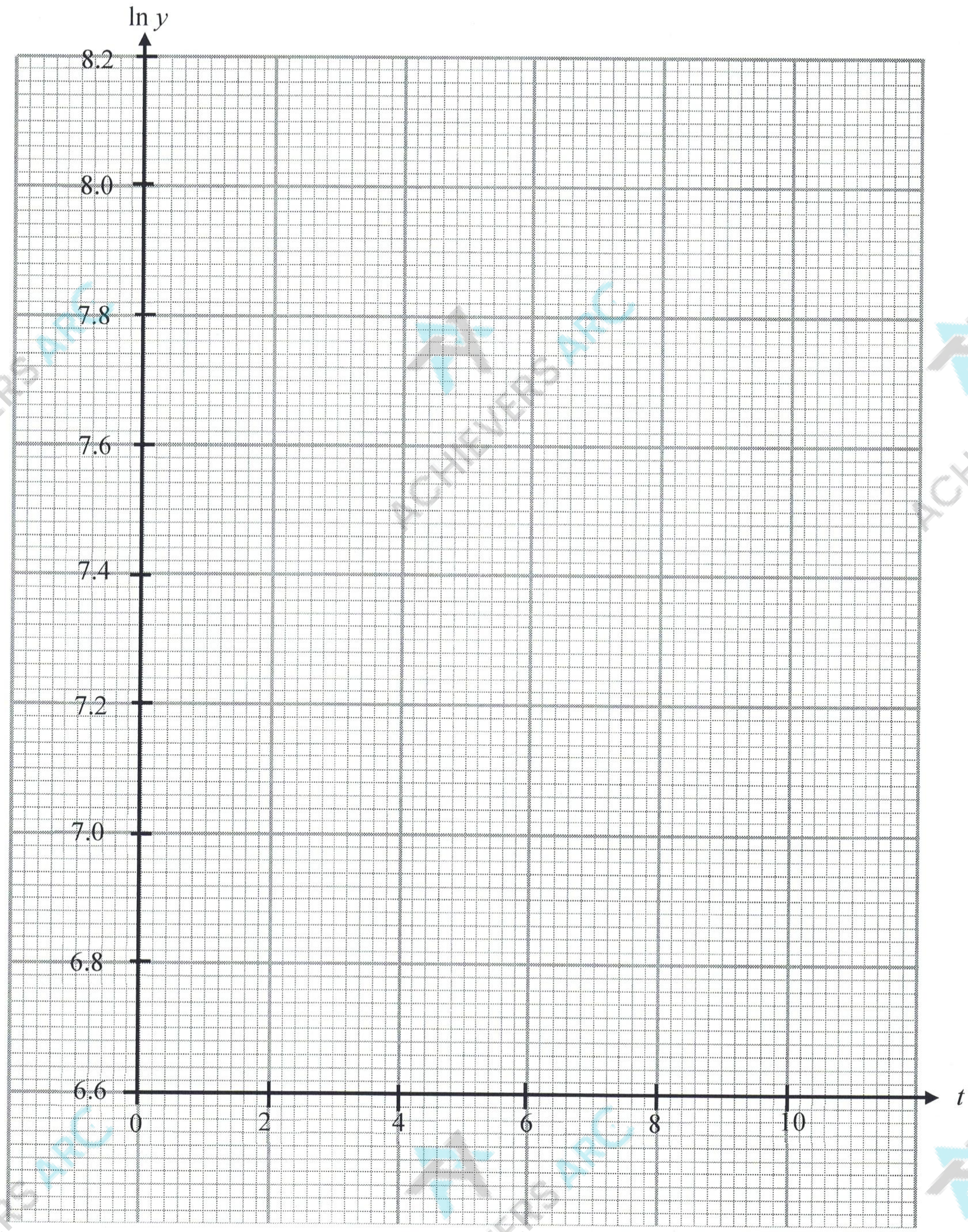
(c) Given that C is a point at which the particle has minimum velocity, determine with working, whether C is nearer to O or to B. [3]

7 A study was performed to assess the number of ants, y, in a colony t weeks after the experiment began.

It is known that the variables y and t can be modelled by the equation $y = ab^{\frac{t}{4}}$, where a and b are constants. The table below shows the values of y and t.

t	2	4	6	8	10
y	1020	1338	1757	2345	3104

(a) Plot $\ln y$ against t on the grid provided and draw a straight-line graph. [2]

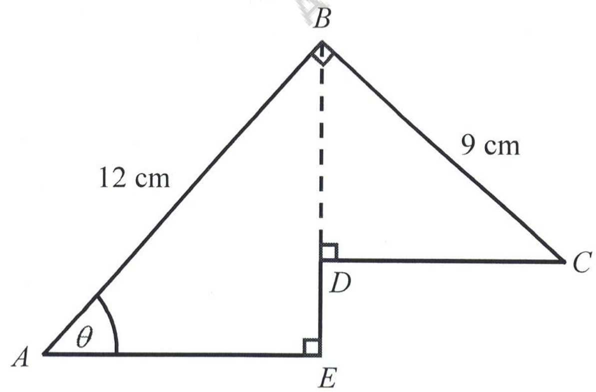


(b) Use your graph to estimate the value of a and of b. [4]

(c) Use your graph to estimate the number of weeks needed for the population of the ants to be doubled from the start of the experiment. [2]

(d) Explain why this model cannot be used five years from the start of the experiment. [1]

8 The diagram shows a figure $ABCDE$ in which $\angle BDC = \angle BEA = \angle ABC = 90°$, $AB = 12$ cm, $BC = 9$ cm and $\angle BAE = \theta$ where $0° < \theta < 90°$.



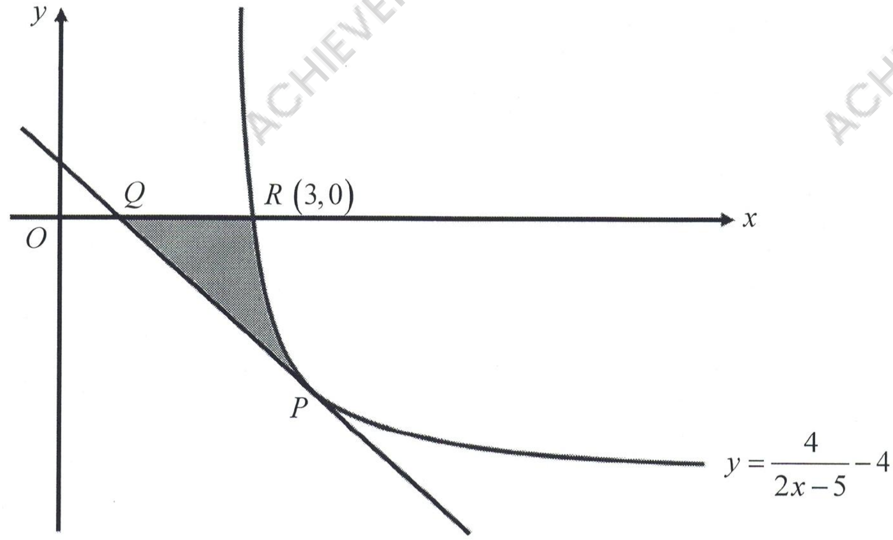
(a) Show that the perimeter, P cm, of the figure is given by

$$P = 21 + 21\sin\theta + 3\cos\theta.$$ [3]

(b) Express P in the form $k + R\cos(\theta - \alpha)$, where k is a constant. [3]

(c) Find the value of θ for which $P = 32$. [2]

9



The diagram shows part of the curve $y = \frac{4}{2x-5} - 4$, cutting the x-axis at $R(3,0)$.

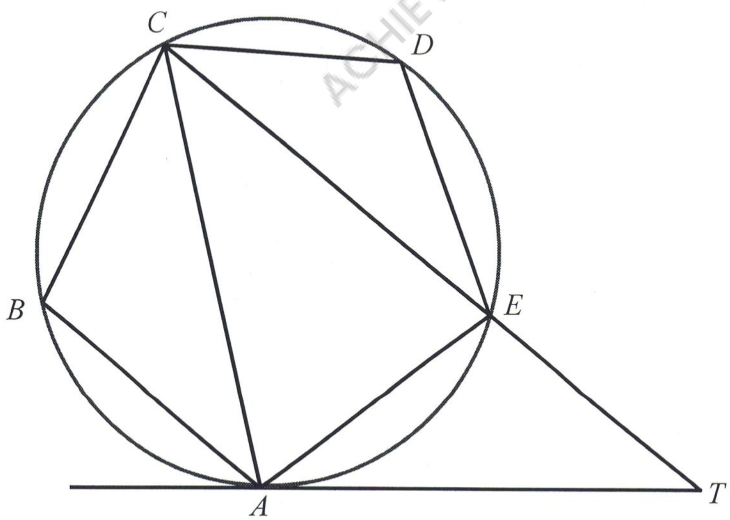
The tangent to the curve at P cuts the x-axis at Q. The normal to the curve at P is parallel to the line $8y = 9x + 16$.

(a) Find the equation of the tangent to the curve at P. [5]

(b) **Without doing any calculation**, explain clearly why $\int_3^4 \frac{4}{2x-5} - 4 \, dx$ is greater than $\int_3^5 \frac{4}{2x-5} - 4 \, dx$. [1]

(c) Calculate the area of the shaded region RPQ. [5]

10

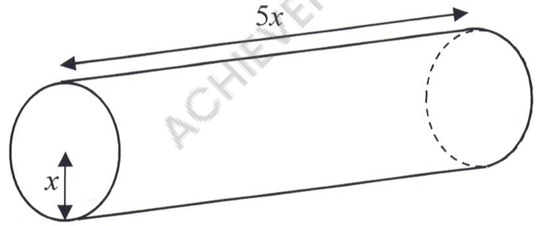


In the diagram, A, B, C, D and E lie on a circle such that CA bisects angle BCE.
The tangent to the circle at A meets CE produced at T. Angle CDE is three times the angle ACB.

(a) Prove that CE is parallel to BA. [5]

(b) Prove that triangle *AET* is an isosceles triangle. [3]

11



The diagram shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm. The cross-sectional area of the rod is increasing at a constant rate of $0.035\ \text{cm}^2$ per second.

(a) Given that the rate of increase of the radius of the rod is 0.005 cm per second, find the length of the radius at this moment. [3]

(b) Find the rate of increase of the radius of the rod when the volume of the rod is $135\pi\ \text{cm}^3$, giving your answer to 3 significant figures. [4]