9

CHIJ ST. THERESA'S CONVENT
PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/1

Paper 1

27 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **17** printed pages.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

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$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 (a) Solve the equation $\sqrt{x+7} = -x-1$. [3]

(b) Solve the equation $\log_2 x + \dfrac{1}{\log_{16} 2} = \log_2(x+3)$. [3]

2 Express $\dfrac{x^3+4x^2+1}{(x+1)(x-1)}$ in partial fractions. [6]

3 **(a)** Find the values of k for which $x^2 - 4x + k^2$ is always positive for all real values of x. [3]

(b) Show that the line $y = 4x + p$ meets the curve $y = px^2 - 2x - 6$ for all real values of p. [3]

4 **(a)** The curve $y = 2x^2 - 4x - 8$ and the line $y = 4 - 2x$ intersect at the points P and Q.
Find the coordinates of P and of Q. [3]

(b) Express $-2x^2 - 14x + 3$ in the form of $a(x+b)^2 + c$ and hence state the coordinates of the turning point of the curve $y = -2x^2 - 14x + 3$. [4]

5 **(a)** Find the first 4 terms in the expansion of $\left(x^2 - \frac{k}{x}\right)^7$ in descending powers of x.

Give the terms in their simplest form. [3]

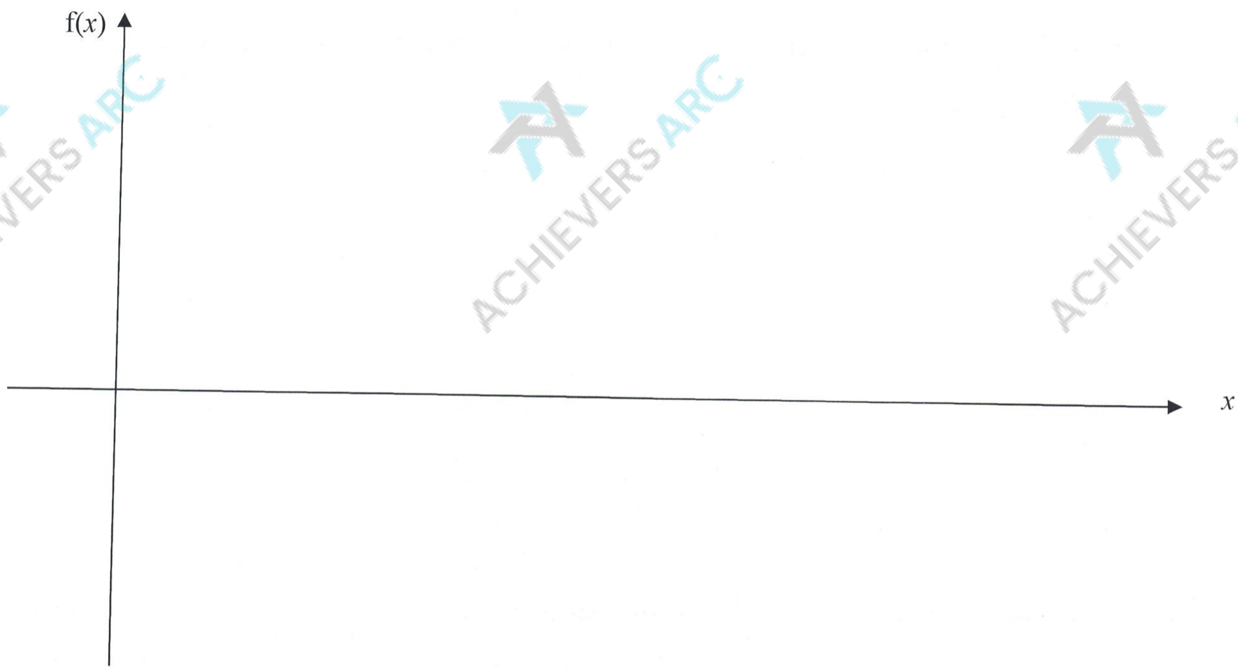
(b) By considering the general term in the binomial expansion of $\left(x^2 - \frac{k}{x}\right)^7$, would there be an x^{-4} term? Justify your answer. [3]

(c) Given that the coefficient of x^{12} in the expansion of $(3x+2)\left(x^2 - \frac{k}{x}\right)^7$ is -105, find the value of the positive constant k. [2]

6 **(a)** Find the amplitude and period of $f(x) = 2 - 3\cos 2x$. [2]

(b) State the least and greatest values of $f(x)$. [1]

(c) Sketch the curve $f(x) = 2 - 3\cos 2x$ for $0 \le x \le 2\pi$. [3]



(d) Hence state the number of solutions of the equation $-3\cos 2x = x$ for $0 \le x \le 2\pi$. [1]

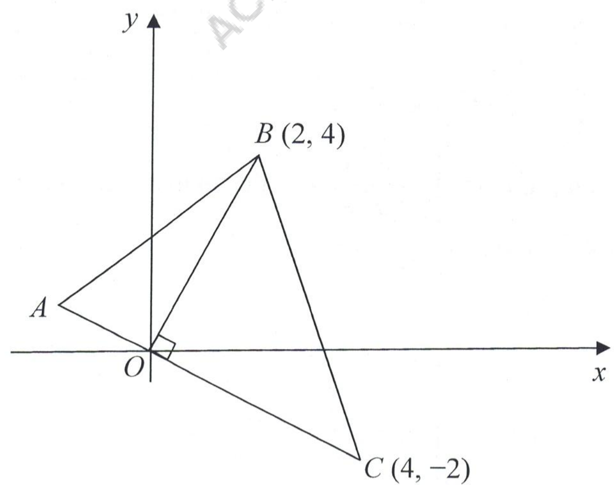
Name:____( ) Class:__________

7 A curve is such that $\frac{d^2y}{dx^2} = 24x^2 + 4e^{2x-1}$.

The curve passes through the point $A\left(\frac{1}{2}, \frac{1}{8}\right)$ and the tangent at A is parallel to the line $y = x + 8$.

Find the equation of the curve. [7]

8



The diagram above shows a triangle ABC in which C is the point (4, –2).
The line AC passes through the origin O. This line OB is perpendicular to the AC.

(i) Find the equation of AC. [1]

(ii) The length of OC is twice the length of OA. Find the coordinates of A. [1]

(iii) Find the area of triangle ABC. [2]

(iv) The point D on OB produced is such that AC is the angle bisector of $\angle DAB$. Find the coordinates of D. [1]

9 A robotic vacuum cleaner is programmed to move back and forth between 2 points A and B along a corridor of a shopping mall while cleaning. It starts cleaning from rest at point A. The speed, v m/s, of the robotic vacuum cleaner is given by $v = \frac{1}{2}\cos\frac{\pi}{3}t$, where t, the time after passing through A, is measured in seconds.

(a) Obtain an expression, in terms of t, for s, the displacement of the robotic vacuum cleaner. [2]

(b) Find the values of t between 0 and 5 at which the robotic vacuum cleaner is instantaneously at rest. [3]

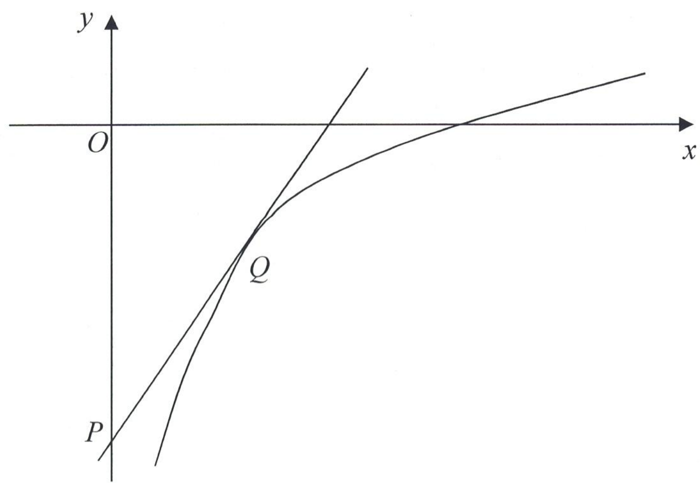
(c) Find the total distance travelled by the robotic vacuum cleaner in the first 3 seconds. [3]

10 It is given that $y = \dfrac{3x^2}{2x-1}$, where $x > \dfrac{1}{2}$.

(a) Find the coordinates of the point on the curve at which the tangent to the curve is parallel to the x-axis. [4]

(b) Find the range of values of x for which y is a decreasing function. [3]

11 The diagram shows part of the curve $y = \dfrac{x-3}{2x-1}$ for $x > \dfrac{1}{2}$.



(a) Find $\dfrac{dy}{dx}$. [2]

(b) Explain why the curve does not have a stationary point. [1]

(c) **(i)** The gradient of the normal to the curve at a point Q is $-\frac{1}{5}$.

The tangent to the curve at Q meets the y-axis at P. Find the equation of the tangent line. [4]

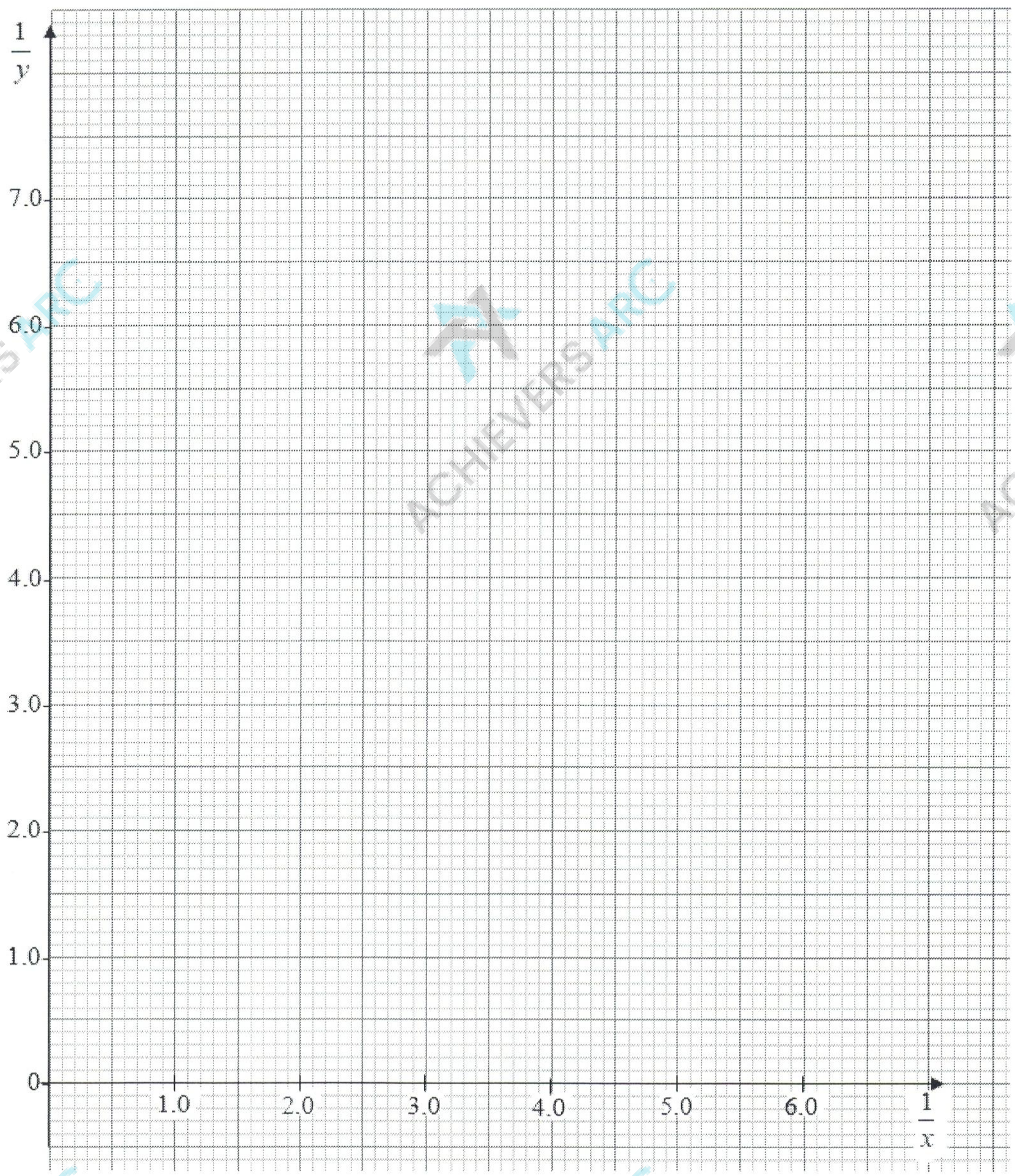
(ii) Find the coordinates of P. [1]

(d) Find the area of the triangle QOP. [1]

12 The table shows experimental values of two variables x and y, which are connected by an equation of the form $\frac{1}{y} = \frac{b}{a}\left(\frac{1}{x}\right) + \frac{2}{a}$, where a and b are constants.

x	0.2	0.4	0.5	0.8
y	1.250	0.285	0.247	0.206

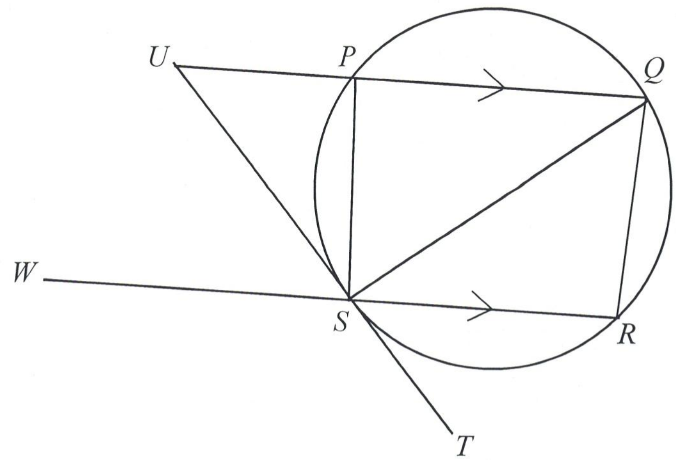
(a) On the grid below, plot $\frac{1}{y}$ against $\frac{1}{x}$ and draw a straight line graph. [3]



(b) Use your graph to estimate the value of a and of b. [4]

(c) Use your graph to solve the equation $\frac{b}{x} + 2 = a$. [2]

13 In the diagram, $PQRS$ is a cyclic quadrilateral in which PQ is parallel to SR.
The tangent to the circle at S meets QP produced at U.
TSU and RSW are straight lines.



(a) Given that angle RQS = angle SUP, show that triangle RSQ and triangle PSU are similar. [3]

(b) If $SR = SU$, show that $SU^2 = PS \times QS$. [2]

~~~ End of Paper ~~~
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CHIJ ST. THERESA'S CONVENT
PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/2**

Paper 2

23 Aug 2024
2 hours 15 minutes

Candidates answer on the Question Paper as well as on the graph paper provided.

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Mathematical Formulae

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1 The equation of a curve is $y = 5\sin^2\left(x - \frac{\pi}{6}\right)$, where $0 \le x \le \frac{\pi}{2}$.

(a) Given that y is decreasing at a rate of 0.3 units per second, find the rate of change of x at $x = \frac{5\pi}{12}$. [3]

(b) The normal to the curve at $x = \frac{5\pi}{12}$ intersects the vertical axis at $(0, k)$.

Find the exact value of k. [3]

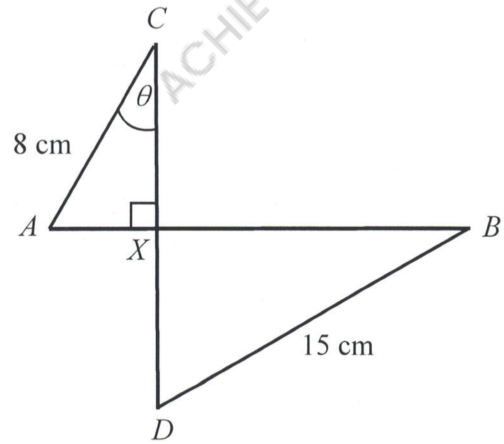
2 **(a)** Find the values of x and y which satisfy the equations

$$8^x - 2^{-y} = 0,$$
$$\left(\sqrt{125^x}\right)^y = \frac{1}{\sqrt{5}}.$$

[3]

(b) Show that the equation $3\left(2^{x+2}\right)-1=35\left(2^{-x}\right)$ has only one solution and find its value correct to 2 significant figures. [5]

3



The diagram shows two perpendicular lines *AB* and *CD* which intersect at *X*. The points *A, B, C* and *D* lie on the circumference of a circle. $AC = 8$ cm, $BD = 15$ cm, and angle *ACD* equals to $\theta°$.

(a) Show that the length of *AB* is $8\sin\theta + 15\cos\theta$. [2]

(b) Express *AB* in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° \le \alpha \le 90°$. [4]

(c) Find the value(s) of θ if $AB = 16$ cm. [3]

4 A calculator must not be used in this question.

It is given that $\frac{\cos(A+B)}{\cos(A-B)} = \frac{1}{3}$.

(a) Show that $\tan A \tan B = \frac{1}{2}$. [3]

(b) If $\tan A = 2+\sqrt{3}$, find an expression for $\tan B$, in the form $a+b\sqrt{3}$, where a and b are constants. [3]

(c) Hence, express $\sec^2 B$ in the form $c + d\sqrt{3}$, where c and d are constants. [3]

5 The equation of a circle is $x^2 + y^2 - 6x + 16y + 48 = 0$.

(a) Find the radius and coordinates of the centre of the circle. [4]

(b) The point $A(0, -4)$ lies on the circle.
Given that AB is a diameter of the circle, find the coordinates of B. [2]

(c) A line with equation $y = mx$, where $m > 0$, does not intersect the circle. A is the point on the circle closest to the line.

(i) Find the value of m. [2]

(ii) Hence, find the coordinates of the point on the line that is closest to the circle. [2]

6 It is given that $f(x) = 4x^p + qx^2 - 3x + 1$, where p and q are constants, has a factor of $x - 1$ and leaves a remainder of -33 when divided by $x + 2$.

(a) Find the values of p and q. [3]

(b) Using the values of p and q found in part **(a)**, solve the equation $f(x) = 0$ completely, leaving non-integer roots in their simplest surd form. [4]

Name: ______________________________ Class: ________ (   )

7 **(a)** Prove the identity $\frac{1-2\cos^2\theta}{\sin\theta\cos\theta} = -2\cot 2\theta$. [3]

(b) Hence, solve the equation $\frac{1-2\cos^2\theta}{\sin\theta\cos\theta} + \tan 2\theta + 1 = 0$, for $0° \le \theta \le 90°$. [5]

8 The blood alcohol concentration, C mg/L, in a person t minutes after he consumes a bottle of wine can be modelled by the formula

$$C = 1250\left(e^{kt} - e^{-0.1t}\right).$$

(a) In Singapore, a driver can be charged with drink driving if he drives when his blood alcohol concentration exceeds 800mg/L. When Jonathan consumes a bottle of wine, his blood alcohol concentration will only fall to 800mg/L after three hours.

Show that $k = -0.0025$ when corrected to 2 significant figures, and find his blood alcohol concentration after 1 hour. [4]

Using $k = -0.0025$,

(b) Find the rate of change of Jonathan's blood alcohol concentration after 1 hour. [2]

(c) After consumption of alcohol, the blood alcohol concentration will rise to a peak before decreasing slowly over time. Explain why the blood alcohol concentration found in part **(a)** is not the peak level, and find the peak blood alcohol concentration level. [5]

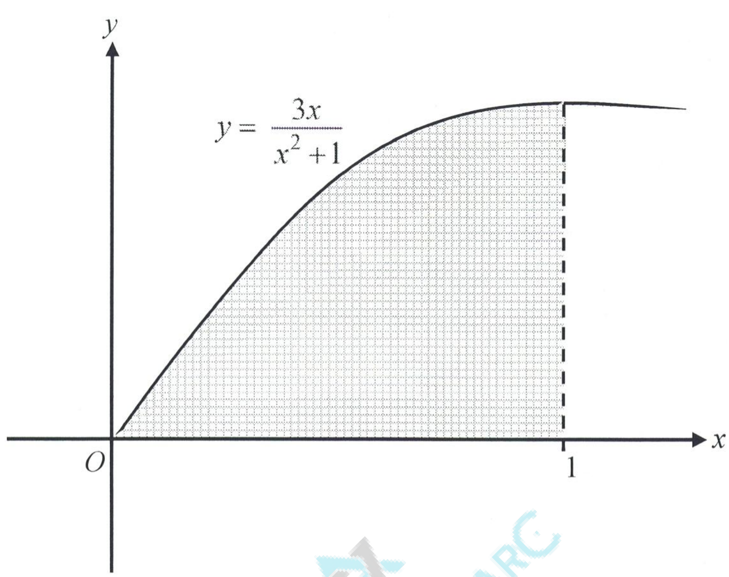
9 **(a)** The equation of a curve is $y = \ln\sqrt{\dfrac{x^2+1}{2x+1}}$.

Show that $\dfrac{dy}{dx} = \dfrac{x}{x^2+1} - \dfrac{1}{2x+1}$. [4]

(b) The diagram below shows part of the graph of $y = \dfrac{3x}{x^2+1}$.

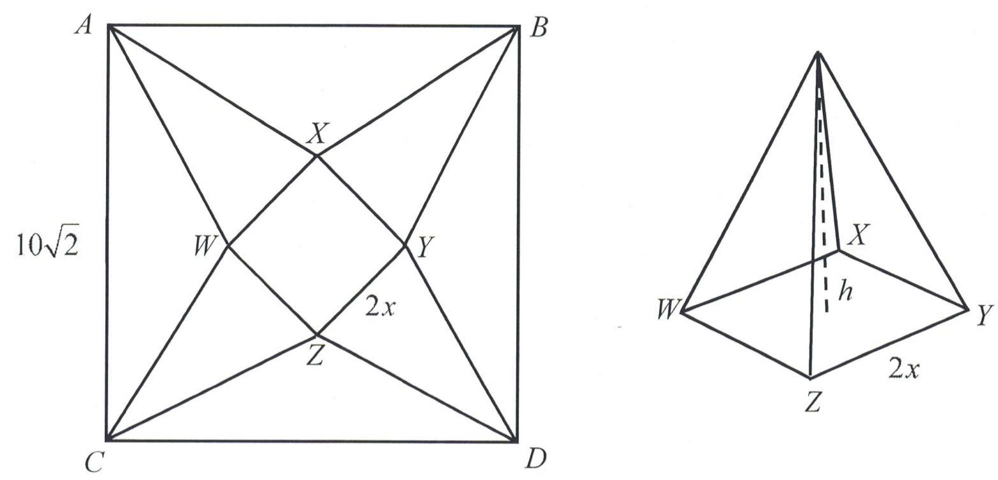
Using the result from part **(a)**, find the area of the shaded region bounded by the curve, the x- axis and the line $x = 1$. Express your answer in the form $a \ln b$, where a and b are constants.

[6]



Continuation of working space for Question 9.

10 In the diagram below, $ABCD$ is a square paper with side $10\sqrt{2}$ cm. The net of a regular pyramid with square base $WXYZ$ was cut from the paper. AB is parallel to WY and the base of the pyramid has sides $2x$ cm.



(a) By expressing the perpendicular height, h cm, of the pyramid in terms of x, show that $V = \frac{8}{3}x^2\sqrt{25-5x}$. [4]

(b) Given that x can vary, find the value of x for which the volume of the pyramid is stationary. [6]

(c) Determine whether this value of x gives a maximum or minimum value for the volume of the pyramid. [2]

~~~ **End of Paper** ~~~
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