Full Name	Class Index No	Class

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Anglo-Chinese School (Barker Road)

PRELIMINARY EXAMINATION 2024
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS
4049
PAPER 1

2 HOURS 15 MINUTES

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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1 The line $x - y = 3$ intersects the curve $x^2 - 3xy + y^2 + 19 = 0$ at the points P and Q.
Find the coordinates of P and of Q. [5]

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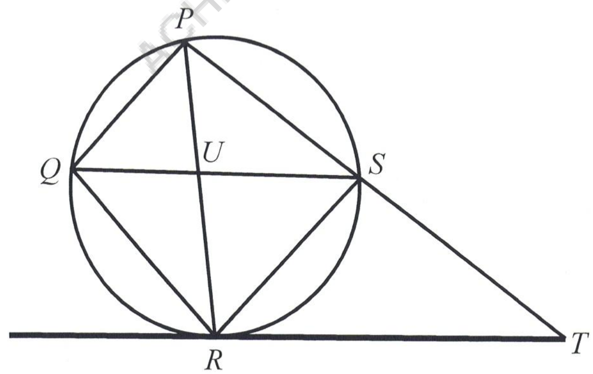
2 Solve the equation $2^{x+1} + 3(2^{-x}) = 7$. [5]

3 Express the equation $\log_2 x + \log_4 (x-1) = 3$ as a cubic equation in x. [4]

4



The diagonals of a cyclic quadrilateral $PQRS$ intersect at U. The tangent to the circle at R meets PS produced at T. If $QR = RS$, prove that

(a) QS is parallel to RT, [3]

(b) triangles QUR and RST are similar. [3]

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TURN OVER FOR QUESTION 5

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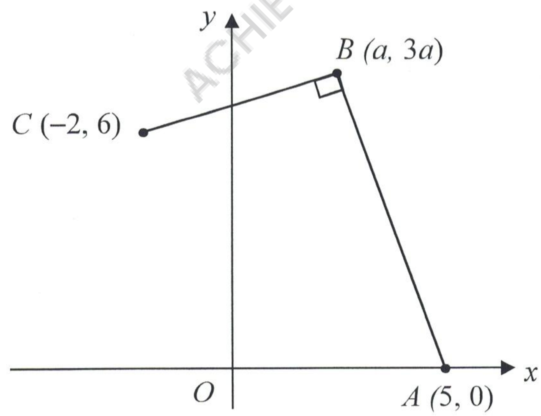

The diagram shows points $A(5, 0)$, $B(a, 3a)$ and $C(-2, 6)$ such that the line AB is perpendicular to the line BC.

(a) Show that $a = 2.5$. [3]

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5 **(b)** Find the coordinates of the midpoint of AC. Hence find the coordinates of D such that $ABCD$ is a parallelogram. [3]

(c) The area of triangle AEC is 5 units^2. Find the value of x given that the point E is $(x, 2x)$, where $x > 1$. [2]

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6 **(a)** Express $\dfrac{13x-6}{x^2(2x-3)}$ in partial fractions. [5]

6 **(b)** Hence $\int \frac{13x-6}{x^2(2x-3)}\,dx$. [3]

7 **(a)** **(i)** Write down and simplify, in ascending powers of x, the first four terms of the expansion of $(1-x)^{12}$. [2]

(ii) Hence find the value of p given that the coefficient of x^3 in the expansion of $(2x^2+17x+p)(1-x)^{12}$ is 6598. [3]

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7 **(b)** In the binomial expansion of $\left(x+\dfrac{k}{x}\right)^5$, where k is a positive constant, the coefficients of x^3 and x are the same. Find the value of k. [4]

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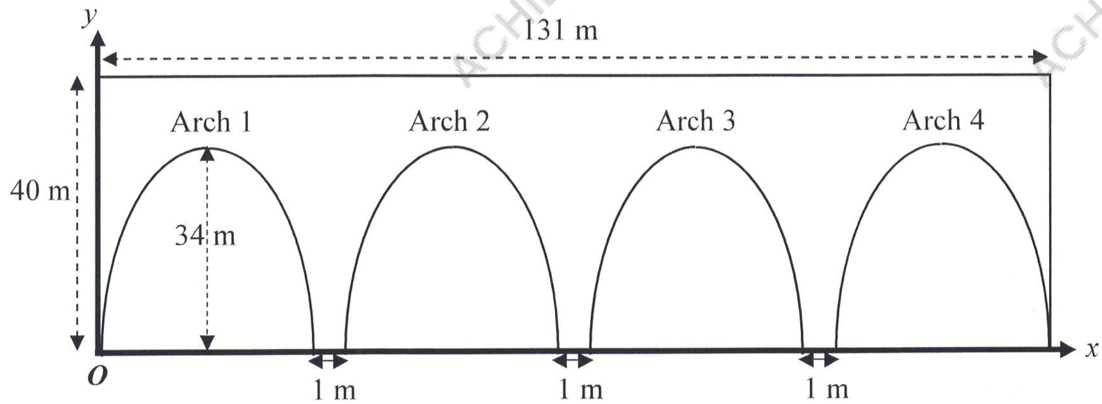

A civil engineer is designing a bridge which is 131 metres long, 40 metres high and is to have four identical parabolic arches along its length. Each arch is 34 metres high and there is one metre between bases of each adjacent pair of arches as shown in the diagram. A set of axes is placed with the origin at the left- hand end of the base of the first arch.

(a) Find the x-intercepts of Arch 1. [2]

(b) The equation representing Arch 1 can be written in the form $y = a(x-p)(x-q)$. Show that $a = -\frac{17}{128}$. [2]

8 (c) Explain, with workings, if the point (50, 30) lies on Arch 2. [3]

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9 **(a)** Differentiate $x\sin x+\cos x$ with respect to x. [2]

(b) Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{3}x\sin^3 x\right)=\frac{1}{3}\sin^3 x-x\cos^3 x+x\cos x$. [3]

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9 **(c)** Using the results found in **part (a)** and **part (b)**, find $\int\left(\sin^3 x - 3x\cos^3 x\right)\mathrm{d}x$. [4]

10 **(a)** Prove the identity $\dfrac{\cos x}{\operatorname{cosec} x - 1} + \dfrac{\cos x}{\operatorname{cosec} x + 1} = 2\tan x$. [5]

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10 **(b)** Solve the equation $\tan x + 2\sec^2 x = 5$ for $0 \leq x \leq 2\pi$. [5]

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11 **(a)** **(i)** Find the range of values of x for which $\ln(x^2-3)$ is defined. Leave your answer in surd form. [2]

(ii) Given that $y=\ln\left[e^x(x^2-3)\right]$, show that $\frac{dy}{dx}$ can be expressed in the form of $\frac{(x+a)(x+b)}{x^2-3}$. [3]

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11 **(b)** It is given that $y = x^3 + px^2 + qx + 10$ where p and q are integers. The only values of x for which y is a decreasing function of x are those values for which $3 < x < 7$. Find the value of p and of q. [4]

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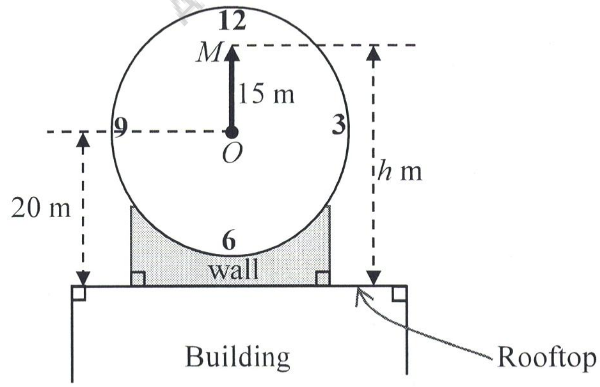

A clock is set on the vertical wall on the roof of a building.
The distance from the centre of the clock, O, to the tip, M, of the minute hand is 15 m.
The height, h m, of M above the rooftop is given by $h = a\cos kt + b$, where t is the time in minutes past the hour. O is 20 m above the rooftop. The rooftop is taken to be horizontal.

(a) Write down the value of a and explain why $b = 20$. [2]

(b) Explain why the value of k is $\frac{\pi}{30}$. [1]

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12 **(c)** Sketch the graph of h for $0 \leq t \leq 45$. [3]

(d) Find the two timings in the first hour for which the height of M is 10 m. [4]

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1 A particle moves along the curve $y=\frac{3(x+6)}{x+4}$, where $x \neq -4$, in such a way that the y-coordinate of the particle is increasing at a constant rate of $\frac{4}{27}$ units per second. Find the x-coordinates of the particle at the instant that the x-coordinate of the particle is decreasing at a rate of 2 units per second. [5]

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2 The number, v, of a certain virus present in a sample collected by a vaccine laboratory is given by $v = me^{2t} + n$, where m and n are constants and t is measured in days. Initially, the number of virus present was 2000. It increased to 5000 after 1 day.

(a) Find the value of m and of n. [4]

(b) Find the number of days in which the number of virus present first reach 1 million. [2]

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3 **(a)** Show that the roots of the equation $ax^2 + (3a+b)x + 3b = 0$ are real for all real values of a and b. [3]

(b) Find the range of values of m for which the line $y = mx - 3$ will never cut the curve $y^2 = 4x - 6y - 34$. [4]

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4 A tangent to a circle at the point (6, 10) passes through the point (9, 6). The centre of the circle lies on the line $3y = 4x + 13$.

Showing all your working, find the equation of the circle. [7]

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5 **Do not use a calculator in this question.**

(a) Express sin 22.5° in the form of $\dfrac{\sqrt{a-\sqrt{a}}}{a}$, where a is an integer. [3]

(b) Show that $\tan 15° = 2-\sqrt{3}$. [5]

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6 (a) (i) Using the substitution $u = x^3$ or otherwise, express $x^6 - 1$ as the product of two factors. [1]

(ii) Hence express $x^6 - 1$ as the product of four factors with integer coefficients. [1]

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6 **(b)** **(i)** Find the remainder when $f(x) = 3x^3 - 5x^2 + 7x - 4$ is divided by $x - 1$. [1]

(ii) Hence show that $h = -1$ for which $g(x) = f(x) + h$ is divisible by $x - 1$. [2]

(iii) Explain why the equation $g(x) = 0$ has only one real root. [4]

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7 **(a)** Two variables x and y are related by the equation $xy^2 = ax + by$. Explain how a straight line graph can be drawn to represent the given equation. [2]

(b) The table shows experimental values of two variables x and y. It is known that x and y are related by the equation $y = pe^{-qx}$ where p and q are constants.

x	1	3	5	7	9
y	98.2	65.9	44.1	29.6	19.8

(i) On the grid below plot $\ln y$ against x and draw a straight line graph. [2]

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7 **(b)** **(ii)** Use your graph to estimate

(a) the value of p and of q, [3]

(b) the value of y when $x = 2$. [1]

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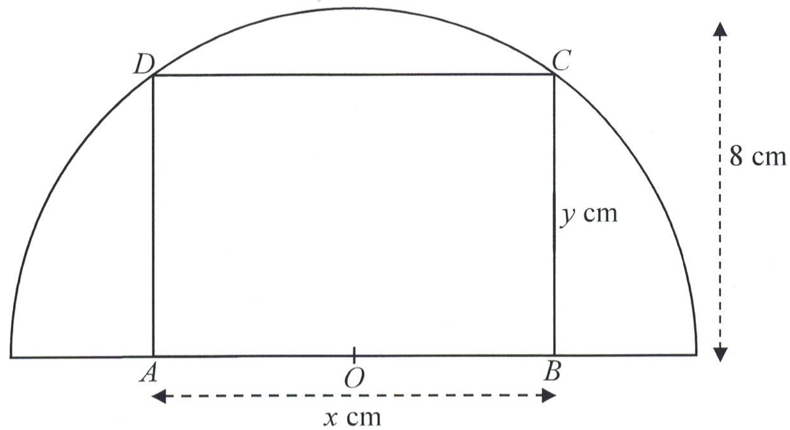
8 $ABCD$ is a rectangle which fits inside a semicircle of radius 8 cm and centre O. It is given that $AB = x$ cm and $BC = y$ cm.



(a) Show that that A cm^2, the area of the rectangle, is given by $A = \frac{x}{2}\sqrt{256 - x^2}$. [2]

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8 **(b)** Given that x can vary, find the value of x which gives a stationary value of A. [4]

(c) By considering the sign of $\frac{dA}{dx}$, determine whether the stationary value of A is maximum or minimum. [2]

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9 A particle starts from rest from a point O and moves in a straight line such that its velocity v m/s, is given by $v = 24t - 6t^2$, where t is the time in seconds after the start of its motion.

(a) Find the value of t at which the particle is instantaneously at rest. [2]

(b) When will the particle return to its starting point? [3]

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9 (c) Determine if the particle is accelerating after 2 seconds. Explain your answer with clear workings. [3]

(d) Calculate the total distance travelled during the first 7 seconds. [4]

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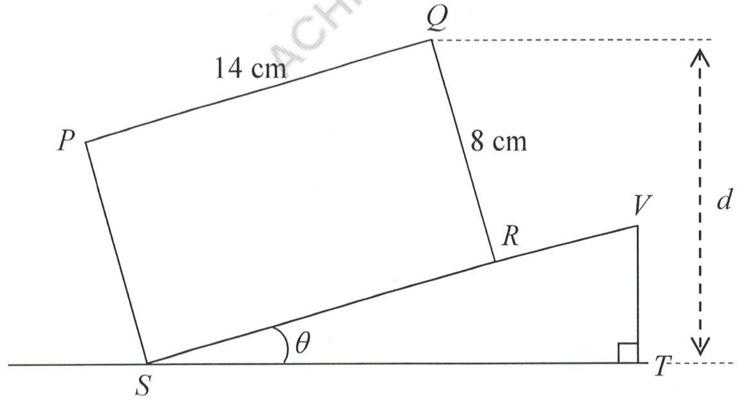

The diagram shows the side view of a 14 cm by 8 cm rectangular block *PQRS*, placed on a ramp, *VS*, tilted at an acute angle of $\theta°$.
The ramp is placed on a horizontal surface *ST* and *d* is the perpendicular distance from *Q* to *ST*. $\angle VTS = 90°$.

(a) Show that $d = 8\cos\theta + 14\sin\theta$. [2]

(b) Express *d* in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

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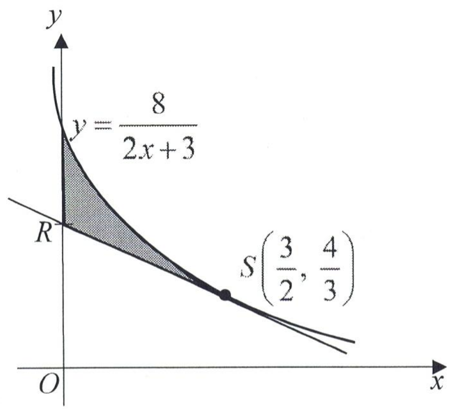
10 **(c)** Find the smallest value of θ such that $d = 10\sqrt{2}$. [4]

11 The diagram shows part of the curve $y = \dfrac{8}{2x+3}$. The tangent to the curve at the point $S\left(\dfrac{3}{2}, \dfrac{4}{3}\right)$ intersects the y-axis at R.



(a) Find the y-coordinate of R. [4]

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11 **(b)** Find the exact area of the shaded region. Express your answer in the form of $\left(\ln a - \dfrac{b}{c}\right)$ units^2, where a, b and c are integers. [6]

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