NAME: ______________________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2024

S4

ADDITIONAL MATHEMATICS **4049/01**
Paper 1
23 August 2024
2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiners' Use

Question	Marks	Question	Marks		
1		8			
2		9		Units	
3		10		Clarity / Logic	
4		11		Precision / Accuracy	
5		12		Total:	
6					
7					90
Parent's Name & Signature:					
Date:					

This paper consists of **17** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

1 Given the curve $y=(a+1)x^2-8x+2a$ has a minimum value, find the range of values of a such that the

(a) line $y=-4ax$ meets the curve. [5]

(b) y-intercept of the curve is greater than 6. [1]

2 The area of a quadrilateral is $\left(21-\sqrt{80}\right)$ cm^2.

(i) In the case where the quadrilateral is a rectangle with width $\left(5-\sqrt{5}\right)$ cm, find, without using a calculator, the length of the rectangle in the form $\left(a+b\sqrt{5}\right)$ cm. [3]

(ii) In the case where the quadrilateral is a square with side $\left(2\sqrt{5}+c\right)$ cm, find, without using a calculator, the value of the constant c. [3]

3 Given that $\frac{dS}{dr} = 8\pi r$ and $\frac{dr}{dt} = \frac{3}{400\pi r^2}$, show that $\frac{dS}{dt}$ is directly proportional to $\frac{1}{r}$. [3]

4 Given that $y = \cos^2 x - \sin^2 x$, show that $\frac{d^2y}{dx^2} = -4y$. [4]

5 **(i)** Find the first 4 terms in the expansion of $\left(2-\frac{ax}{2}\right)^7$ in ascending powers of x, simplifying each term. [4]

(ii) Given that there is no term in x^2 in the expansion of $\left(1+\frac{7x}{2}\right)\left(2-\frac{ax}{2}\right)^7$, find the value of the positive constant a. [2]

(iii) Using this value of a, find the coefficient of x^3 in the expansion of $\left(1+\frac{7x}{2}\right)\left(2-\frac{ax}{2}\right)^7$. [2]

6 Some salmon fillets, stored at a constant temperature in a freezer, was removed for defrosting. The temperature, T °C, of the salmon, t minutes after it have been taken out of the freezer, is modelled by the formula $T = 28 - 46e^{-0.03t}$.

(i) State the temperature of the salmon when they are stored in the freezer. [1]

(ii) Explain, with working, why the temperature of the salmon can never reach 28 °C. [2]

Once the salmon has defrosted, they should be moved to the display fridge at the sushi bar to be served as sashimi.

(iii) Find the value of t, to the nearest whole number, for the temperature of the salmon to drop to 0 °C. [3]

7 **(a)** By using an appropriate substitution, solve the equation $49^x - 18 = 7^{x+1}$.
Leave your answer in the form $b\log_a 3$, where a and b are integers to be determined. [6]

(b) Express the equation $\log_2 x + \log_4 (x+5) = 2$ as a cubic equation in x. [5]

8 Given that $y = \dfrac{2x}{\sqrt{x-1}}$,

(i) find $\dfrac{dy}{dx}$, [3]

(ii) hence, find the range of values of x for which y is decreasing. [3]

9 A circle C_1 has centre $A(4,2)$ and radius $\sqrt{13}$ cm.

(i) Write down the equation of the circle C_1. [1]

(ii) Determine whether the point $R(8,3)$ lies inside or outside the circle C_1. [2]

The circle C_1 intersects the x-axis at points P and Q.

(iii) Find the mid-point of PQ. [4]

A second circle C_2, with centre B and radius $\sqrt{18}$ cm, also passes through points P and Q.

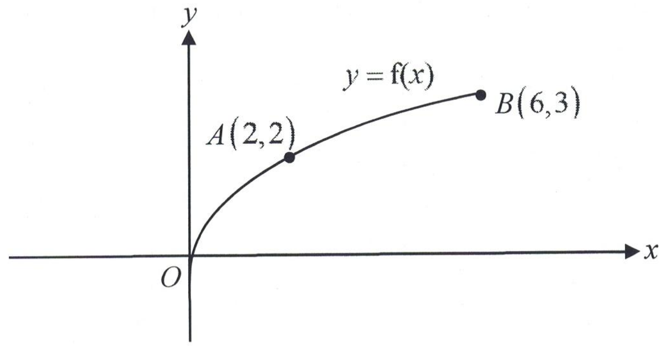
(iv) State the x-coordinate of point B. [1]

(v) Given point B lies above the x-axis, find the y-coordinate of point B. [2]

10 The equation of a curve is $y = x\ln(x - \mathrm{e})$.
The curve meets the line $y = x$ at point P.
Show that the equation of the normal to the curve at point P is $ay = b\mathrm{e} - x$, where a and b are constants to be determined. [7]

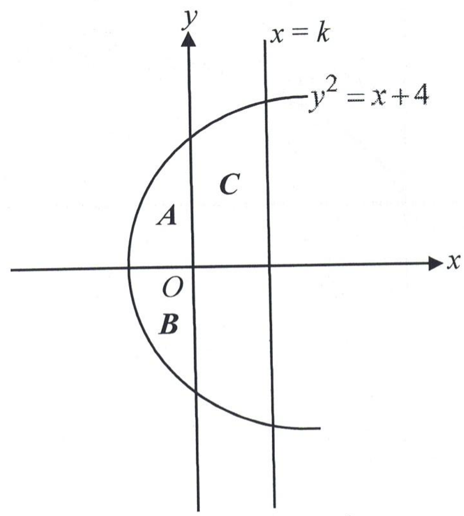
11 **(a)** In the diagram below, $A(2, 2)$ and $B(6, 3)$ lie on the curve $y = \mathrm{f}(x)$.

Find the value of $\int_2^3 x\,\mathrm{d}y + \int_2^6 y\,\mathrm{d}x$. [2]

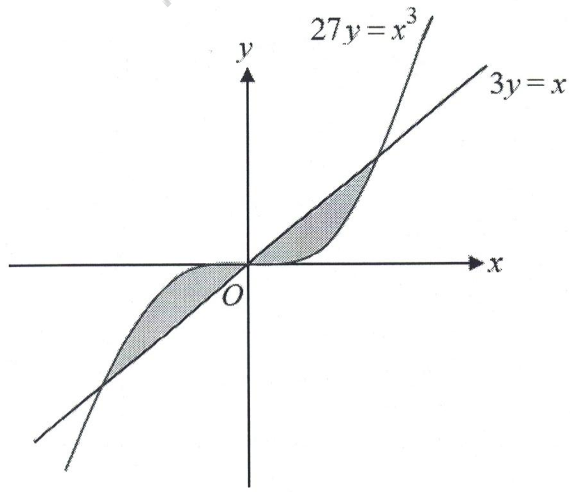


(b) The diagram shows part of the curve $y^2 = x + 4$ and the line $x = k$, where k is a constant. The area of the region A and B equals to the area of the region C.

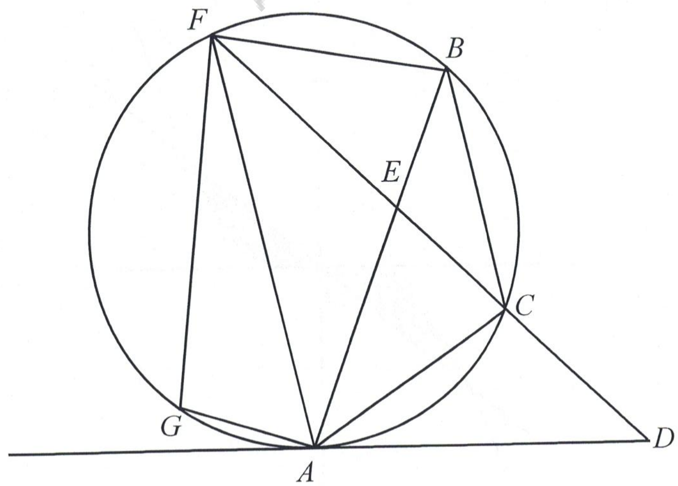
Show that the value of k equals to 4.32, when correct to three significant figures. [7]



(c) The diagram below shows the shaded region enclosed by the curve $27y = x^3$ and the line $3y = x$. Find the area of the shaded region. [5]



12 The diagram shows a circle. Vertices of pentagon $ABCFG$ lie on the circle. The line AD is tangent to the circle at A. AC bisects angle BAD. $DCEF$ is a straight line and $AB = AF$.



(i) Prove that triangle ABC is isosceles. [2]

(ii) Prove that triangle DAC is similar to triangle DFA. [2]

(iii) Show that $BC \times FD = AD \times AB$. [2]

(iv) Show that angle AGF + angle DAC + angle $ADC = 180^\circ$. [3]

END OF PAPER

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ANGLICAN HIGH SCHOOL
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PRELIMINARY EXAMINATIONS 2024

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ADDITIONAL MATHEMATICS | **4049/02**
Paper 2

27 August 2024
2 hours 15 minutes

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Additional Material: Graph Paper. (1 sheet)

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1 Given that $(2x+1)$ is a factor of $f(x)=2x^3+9x^2+ax+7$.

(i) Find the value of a. [2]

(ii) Hence, prove that there is only one solution for which $f(x)=0$. [3]

2 *PQRS* is a trapezium with *PS* parallel to *QR*.
The coordinates of *S* are $(3,\ 10)$ and the equation of *PS* is $y = 2x + 4$.
Given that *PQ* is parallel to the line $7y + 4x - 7 = 0$ and that *P* lies on the *y*-axis and *Q* lies on the *x*-axis, find

(i) the coordinates of *P* and *Q*, [4]

(ii) the coordinates of *R* given that the line $y = x$ passes through *R*, [3]

(iii) area of the trapezium *PQRS*. [2]

(iv) Hence, using the result in **(iii)**, show that the perpendicular distance between the lines PS and QR can be written as $k\sqrt{5}$ where k is a constant. [3]

3 The function $\mathrm{f}(x)$, for $0 \le x \le 2\pi$, is defined by $\mathrm{f}(x) = a\cos bx + c$ where a, b and c are integers.
Given that $\mathrm{f}(x)$ has a maximum value of 12 and a minimum value of −2,

(i) state the value of c, [1]

(ii) state the possible values of a. [2]

Given further that the period of $\mathrm{f}(x)$ is $\frac{\pi}{3}$,

(iii) state the value of b, [1]

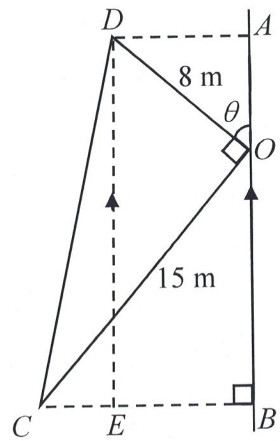
4 By writing $15°$ as a difference of two angles, show that $\cot 15°$ can be expressed as $a+b\sqrt{3}$, where a and b are integers. [5]

5 **(i)** Show that $\dfrac{\sec x+\operatorname{cosec} x}{\tan x+\cot x}=\sin x+\cos x$. [3]

(ii) Hence solve the equation $\left(\dfrac{\sec x+\operatorname{cosec} x}{\tan x+\cot x}\right)^2=\sin^2 x$ for $0<x<2\pi$. [5]

6 In the diagram, $OD = 8\,\text{m}$, $OC = 15\,\text{m}$ and angle COD = angle OAD = angle $CBO = \frac{\pi}{2}$.

The point E lies on the line BC such that DE is parallel to AB.



(i) Given that angle $DOA = \theta$ where $0 < \theta < \frac{\pi}{2}$, show that $AB = 15\sin\theta + 8\cos\theta$. [2]

(ii) Express AB in the form $R\sin(\theta + \alpha)$ where $R > 0$ and α is an acute angle. [2]

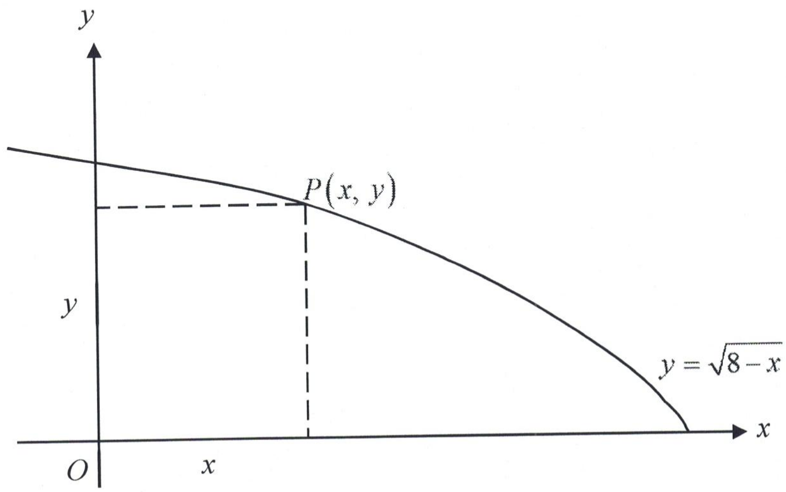
(iii) From the diagram, state which line has a length R m and which angle has a value of α. [2]

7 **(a)** The volume V of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

If the volume of a spherical balloon is increasing at a constant rate of 1 m^3/s, find the volume of the balloon when the rate of increase of its radius is $\frac{1}{\pi}$ m/s. [3]

(b) Variables x and y are related by the equation $y = xe^{2-x}$.
Given that y is decreasing at a constant rate of 2 units per second at $x = 3$, find the rate of change in x, leaving your answer in terms of e. [4]

8 The diagram shows part of the graph of $y = \sqrt{8-x}$ for $0 < x < 8$.
O is the origin and $P(x, y)$ is a point on the curve such that O and P are opposite vertices of the rectangle shown. Given that x and y can vary,



(a) write down an expression for the area of the rectangle, A units^2, in terms of x and show that $\frac{dA}{dx} = \frac{16-3x}{2\sqrt{8-x}}$. [3]

(b) Hence, using the result in **(a)**, find the maximum area of the rectangle, correct to 1 decimal place. [5]

9 **(a)** Find the indefinite integral $\int \frac{x\left(1+x^2\right)}{x^n}\,\mathrm{d}x$, where n is a positive integer. [3]

(b) Integrate $1-\sin 2x+\tan^2 x$ with respect to x. [3]

(c) Find $\int \frac{3}{4\mathrm{e}^{3x-6}}\,\mathrm{d}x$. [2]

10 **(a)** It is given that $\int_2^4 p(3x-8)^4 \; dx = 1760$ where p is a constant.
Find the value of p. [3]

(b) Show that $\frac{d}{dx}\left[\ln\left(\frac{x}{2x+1}\right)\right] = \frac{1}{x(2x+1)}$. [2]

11 **(i)** Express $\dfrac{x+1}{x(2x+1)}$ in partial fractions. [3]

(ii) Hence, using the result in **(i)** evaluate $\displaystyle\int_1^2 \frac{4x^2+3x+1}{x(2x+1)}\,\mathrm{d}x$.

Leave your answer correct to 2 decimal places. [5]

12 An electron moves in a straight line such that at time t seconds after leaving a fixed point P, its velocity v m/s is given by $v = 15 - 6e^{2t}$.

Find

(a) the time the electron comes to instantaneous rest, [2]

(b) the total distance travelled by the electron in the first second. [4]

(c) the acceleration when $t = \ln 2$. [2]

13 **Answer the whole question on a sheet of graph paper.**

The variables x and y are related by the equation $y = ax^{b+1}$, where a and b are constants. The table shows the experimental values of x and y.

x	1.5	2	3	4	5	6
y	6.7	5.0	3.3	2.5	2.0	1.7

(i) Plot $\lg y$ against $\lg x$ and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of a and of b. [2]

(iii) By drawing a suitable line on your graph, solve the equation $ax^b = 1$. [2]

END OF PAPER