AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2024

SECONDARY 4 EXPRESS

Name:	Class:	Register No.:
MARKING SCHEME		

ADDITIONAL MATHEMATICS
Paper 1

4049/01
5 August 2024

Candidates answer on the Question Paper.

2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
/90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
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$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1	**(a)**	Factorise $\sin^3 x - \cos^3 x$ and leave your answer in the form $(\sin x - \cos x)(p + q\sin kx)$, where p, q, k are constants to be determined. [3]
	(b)	Integrate $\tan^2 x$ with respect to x. [2]

2	The curve $y = \log_a x$ passes through the points $(8, 3)$ and $(c, -2)$.	
	(a)	Determine the values of a and c. [2]
	(b)	Sketch the graph of $y = \log_a x$. [2]

3	A curve has the equation $y = 3-(5-x)^2$. The point P is a stationary point on the curve.
	(a) State the coordinates of point P and explain why it is a maximum point. [2]
	(b) A straight line has the equation $y = mx+9$. Find the range of values of m for which the curve lies completely below the line. [3]

4	**(a)**	Given that $u = 12^x$, express $36\left(144^x\right) + 23\left(3^{x+1}\right)\left(2^{2x}\right) = 6$ as an equation in u. [3]
	(b)	Explain why $36\left(144^x\right) + 23\left(3^{x+1}\right)\left(2^{2x}\right) = 6$ has only one solution and find its value. [3]

5	**(a)**	Solve the equation $x+2\sqrt{x-3}=11$. [4]
	(b)	Given that $p=3+2\sqrt{3}$, express $\dfrac{p^2}{p-6}$ in the form $a\sqrt{3}+b$ where a and b are integers to be determined. [4]

6	**(a)** Express $\dfrac{2x^2-4-x^3}{x(x+2)^2}$ in partial fractions.	[6]
	(b) Find $\displaystyle\int \frac{2x^2-4-x^3}{3x(x+2)^2}\,\mathrm{d}x$.	[3]

7	The equation of a curve is $y = \ln\sqrt{5-2x}$.
	(a) A particle moves along the curve such that at the point T, the x-coordinate of the particle is increasing at 0.2 units/sec and the y-coordinate is decreasing at 0.05 units/sec. Find the coordinates of T. [4]
	(b) The normal to the curve at the y-intercept meets the x-axis at A and the y-axis at B. Find the area of triangle AOB, where O is the origin. [5]

8 A circle, C_1, has equation $x^2 + y^2 - 16x + 8y + 64 = 0$.

(a) Find the radius and the coordinates of the centre of C_1. [3]

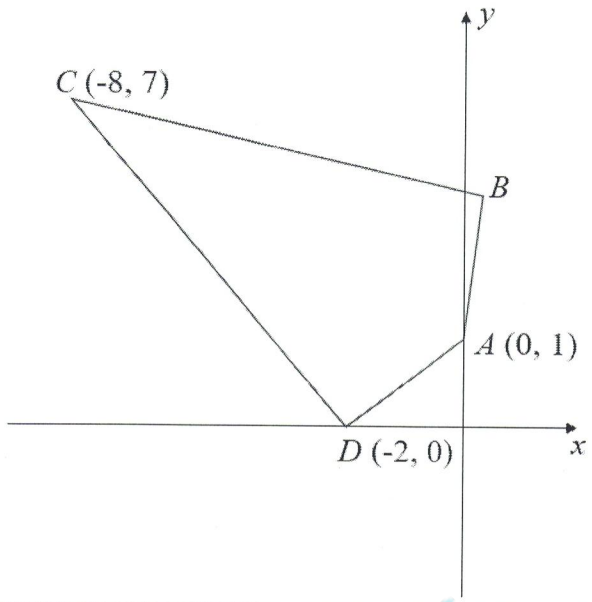
(b) The line $y = k$ is a tangent to the circle at point P, where $k \neq 0$. Find the value of k. [1]

(c) The tangent to the circle at the point $Q(4, -4)$ intersects $y = k$ at the point R. State the equation of this tangent. [1]

(d) Explain why a circle C_2 can be drawn through the points P, Q and R with PQ as the diameter. [2]

(e) Find the equation of C_2. [3]

9 The diagram shows a kite $ABCD$ such that A, C and D are the points $(0,1)$, $(-8,7)$ and $(-2,0)$ respectively.



(a) Find the equation of the line AC. [2]

(b) Find the equation of the line BD. [2]

(c) Find the coordinates of the point B. [5]

(d) Find the area of the kite $ABCD$. [2]

10 **(a)** Solve the equation $3\cos 2A + \sin A + 2 = 0$ for $-180° \leq A \leq 180°$. [5]

(b) On the same axes, sketch the graphs of $y = 1 - \frac{3}{2}\cos 4x$ and $y = 2 + \frac{1}{2}\sin 2x$ for $-90° \leq x \leq 90°$. [5]

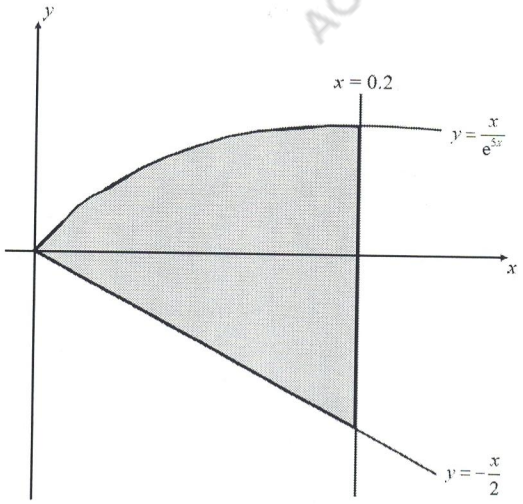
	(c) Explain how the solutions of the equation in **part (a)** could be used to find the x - coordinates of the points of intersection of the graphs of **part (b)**. [2]

11 **(a)** Differentiate $\frac{x}{e^{5x}}$ with respect to x. [2]

(b) Find the obtuse angle between the tangent to the curve $y = \frac{x}{e^{5x}}$ at the point $x = 0$ and the x – axis. [2]

(c) Use the result from **part (a)** to show that $\int_0^{0.2} \frac{x}{e^{5x}} \, dx = \frac{e-2}{25e}$. [4]

(d)



The diagram above shows part of the curve $y = \frac{x}{e^{5x}}$ and the lines $y = -\frac{x}{2}$ and $x = 0.2$. Find the area of the shaded region bounded by the curve and the lines.

[3]

END OF PAPER

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1	(a)	Every term in the expansion of $(x+\frac{1}{2x^2})^n$ is **dependent** on x. By considering the general term in the binomial expansion of $(x+\frac{1}{2x^2})^n$, write down one comment on the possible values of n.	[3]
	(b)	Find the value of n if the ratio of the coefficients of the fourth term to the third term is $5:2$.	[3]

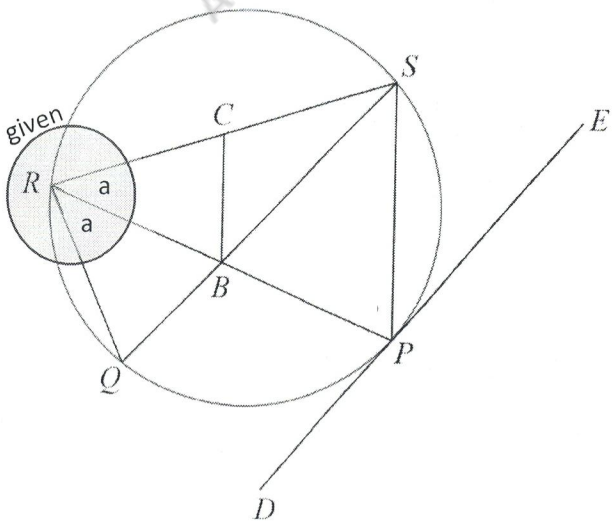
2	Since the start of 2023, the chicken population, P, on a farm has been steadily decreasing. The farm claims that this decrease is exponential and can be modelled by the equation $P = 5000(1+4e^{-0.025t})$, where t is the time in years after 2023.		
	(a)	State the number of chickens on the farm at the start of 2023.	[1]
	(b)	Explain why the population of the chicken on the farm can never reach 5000.	[1]
	(c)	Sketch the graph of P against t to illustrate the population of chicken on the farm from 2023.	[2]
	(d)	Determine the year in which the population of the chicken on the farm first dropped below half of its 2023 population.	[3]

3	$f(x)$ is such that $f''(x) = 4\cos 4x + 2\sin 2x$. Given also that $f(0) = 0$ and $f\left(\frac{\pi}{4}\right) = \frac{3}{4}$, show that $f\left(\frac{\pi}{6}\right) = \frac{7-2\sqrt{3}}{8}$.	[8]

4	**(a)**	Show that $\cos(A+B)\cos(A-B) = \cos^2 A + \cos^2 B - 1$.	[3]
	(b)	Without using a calculator, deduce the value of $\cos\frac{\pi}{12}\cos\frac{5\pi}{12}$.	[5]

	(c)	Write down the principal value of the following in terms of π.	
		(i) $\cos^{-1}\left(-\frac{1}{2}\right)$	[1]
		(ii) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$	[1]

5



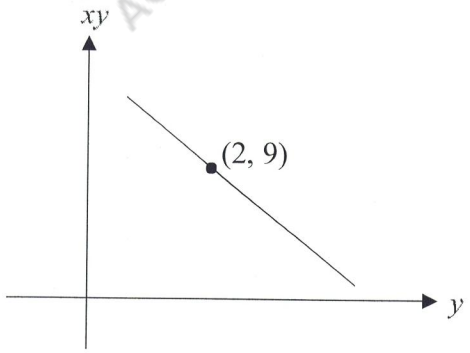
The diagram shows a circle passing through the points P, Q, R and S.
DPE is a tangent to the circle at point P and angle BRC = angle BRQ.
B and C are the midpoints of PR and SR respectively.

(a) Prove that angle SPE = angle BRQ. [2]

(b) Prove that triangle BRC is similar to triangle QRB. [4]

	(c)	Show that $BR \times BQ = \frac{1}{2} PS \times QR$.	[2]

6	**(a)**	The diagram shows part of a straight line graph drawn to represent the equation $y = \frac{a}{x-b}$, where a and b are constants. Given that the line passes through (2, 9) and has a gradient –2, find the value of a and of b.	[3]



(b) The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y = ax^b$, where a and b are constants.

y	2.45	2.94	3.69	4.30	4.88
x	2	3	5	7	10

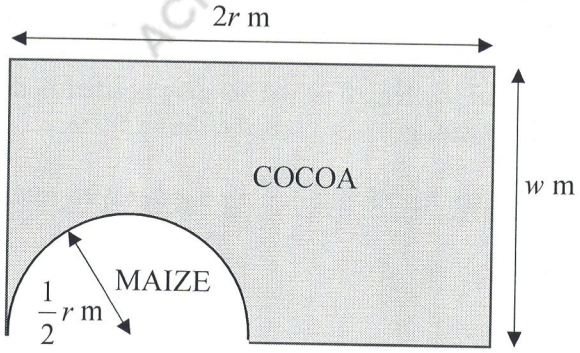
(i) On the grid next page, plot $\ln y$ against $\ln x$ for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

7	A particle moves in a straight line so that, t s after passing through a fixed point O, its velocity, v m/s, is given by $v = 2t - 11 + \frac{6}{t+1}$.		
	(a)	Find the acceleration of the particle when the particle is at instantaneous rest.	[5]

	(b)	Find the distance travelled by the particle in the sixth second.	[2]
	(c)	Find the total distance travelled by the particle in the first 6 seconds.	[4]

8



The diagram shows a rectangular field measuring $2r$ m by w m.

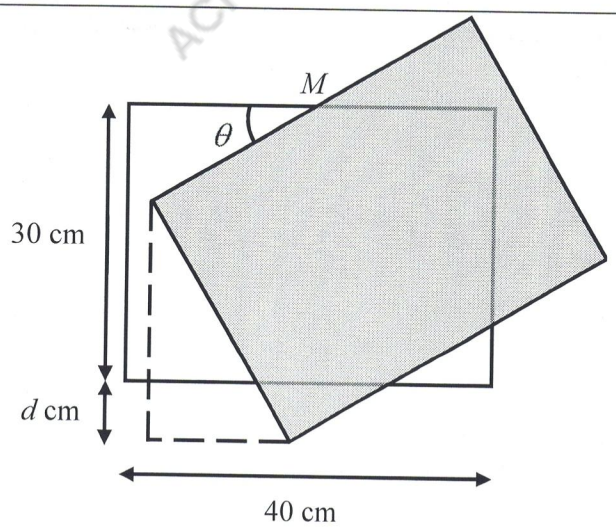
A semicircular plot of radius r m is to be planted with maize while the rest of the field is to be planted with cocoa trees.

(a) Given that the area of the field for the cocoa trees is 200 m^2, express w in terms of r. [2]

(b) Given that the perimeter of the field for the cocoa tress is P m, show that $P = \left(\frac{5\pi}{8} + 3\right)r + \frac{200}{r}$. [2]

	(c)	The farmer wants to put a fence along the perimeter of the field for the cocoa trees to better protect it against strong winds. She claimed that the length of the fencing needed is the least when the maize crop occupies a semicircular plot of radius 5.09 m. Do you agree with her? Explain your answer with relevant workings.	[5]

9



The diagram shows a rectangular picture frame 40 cm by 30 cm hung on the wall. The picture frame is rotated through an angle about the midpoint, M of the top edge.

(a) Show that the vertical displacement, d cm, of the picture frame below its original bottom edge is given by $d = 20\sin\theta + 30\cos\theta - 30$. [2]

(b) Express d in the form $R\sin(\theta + \alpha) - 30$, where $R > 0$ and $0° \leq \alpha \leq 90°$. [4]

	(c)	Find the value of d and the corresponding value of θ that will give the greatest vertical displacement of the picture frame below its original bottom edge.	[3]

10	The equation of a polynomial is given by $f(x) = 4x^3 + 9x - 5$.		
	(a)	Find the remainder when f(x) is divided by $x + 1$.	[1]
	(b)	Show that $2x - 1$ is a factor of $f(x)$.	[1]
	(c)	Show that the equation f(x) = 0 has only one real root.	[4]

	(d)	Hence, or otherwise, solve the equation $\frac{4}{3}\left(3^{3y+1}\right)+9\left(3^{y}\right)=5$.	[4]
	(e)	Show that the solution in **part (d)** may be written in the form $a-\log_b 2$, where a and b are integers to be determined.	[2]

END OF PAPER