Candidate Name	Form Class	Index Number

ANG MO KIO SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2024
SECONDARY THREE EXPRESS

MATHEMATICS
Paper 1

4052/01
03 October 2024
1 hour 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
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The total of the marks for this paper is 60.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use
60

This document consists of **14** printed pages.

Mathematical Formulae

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$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curve surface area of a cone $= \pi rl$

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Answer **all** the questions.

1 Evaluate $\dfrac{0.27^3}{1+\sqrt{4.208}}$, giving your answer correct to 4 significant figures.

Answer ..... [1]

2 **(a)** The diameter of a water molecule is 0.28 nanometres.
Express 0.28 nanometres in centimetres, giving your answer in standard form.
[1 nanometre = 10^{-9} metres]

Answer ..... cm [1]

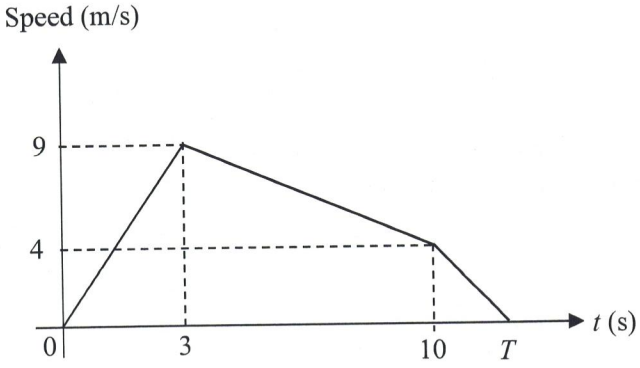
(b) The water molecules are placed side by side to form a line segment of 5 cm. Find the number of water molecules on the line segment, expressing your answer in standard form.

Answer ..... [2]

3 Andrew invested \$15000 in a bank at an interest rate of r % per annum. The bank pays an interest compounded half-yearly. At the end of 3 years, the total interest earned is \$1872.96. Find the value of r.

Answer $r =$ ..... [3]

4 The diagram below shows a speed-time graph of a moving particle over a period of t seconds. It is given that the area under a speed-time graph represents the distance travelled.



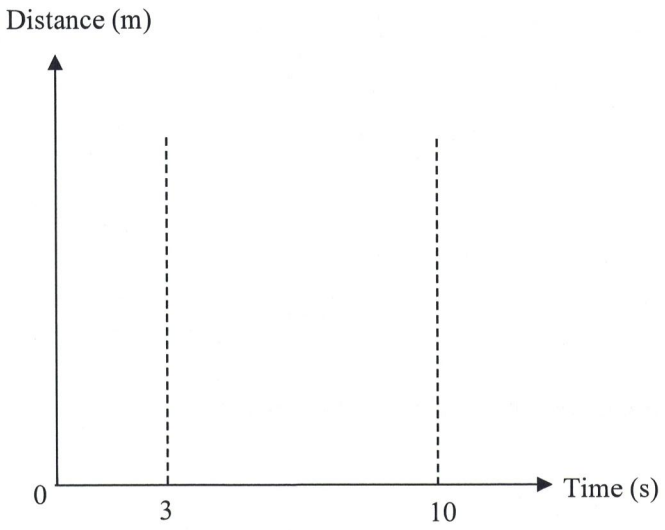
(a) Find its speed when $t = 5$ seconds.

Answer ..... m/s [2]

(b) The distance travelled from 3 seconds to T seconds is 49.5 m. Find the value of T.

Answer $T =$ ..... [2]

(c) On the answer space provided below, sketch the distance-time graph for the first 10 seconds.



[2]

---

5 **(a)** Expand and simplify $y^2 - \left(y - \frac{3}{y}\right)^2$.

Answer ..... [2]

(b) Factorise completely $6x^2y - 1 + 3x - 2xy$.

Answer ..... [2]

---

6 At 0700, a hotel starts to drain its outdoor swimming pool at a constant rate.
The pool is left with $\frac{3}{4}$ of the water at 0930. Find

(a) the fraction of the water drained in 1 hour,

Answer ..... [1]

(b) the time when the pool is left with $\frac{1}{5}$ of the water.

Answer ..... [2]

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7 The force of attraction, F newtons, between the magnets is inversely proportional to the square of the distance, d cm, between the magnets. If the distance is reduced by half its original value, calculate the percentage increase in the force of attraction between the magnets.

Answer ..... % [2]

---

8 **(a)** Express as a single fraction in its simplest form.

$$\frac{3x}{4x^2-1}+\frac{5}{3-6x}$$

Answer ..... [2]

(b) Solve the inequalities $-10+x<2x-1\leq\frac{14-x}{2}$.

Answer ..... [2]

---

9 **(a)** Express $x^2 - 2x + 9$ in the form $(x-a)^2 + b$.

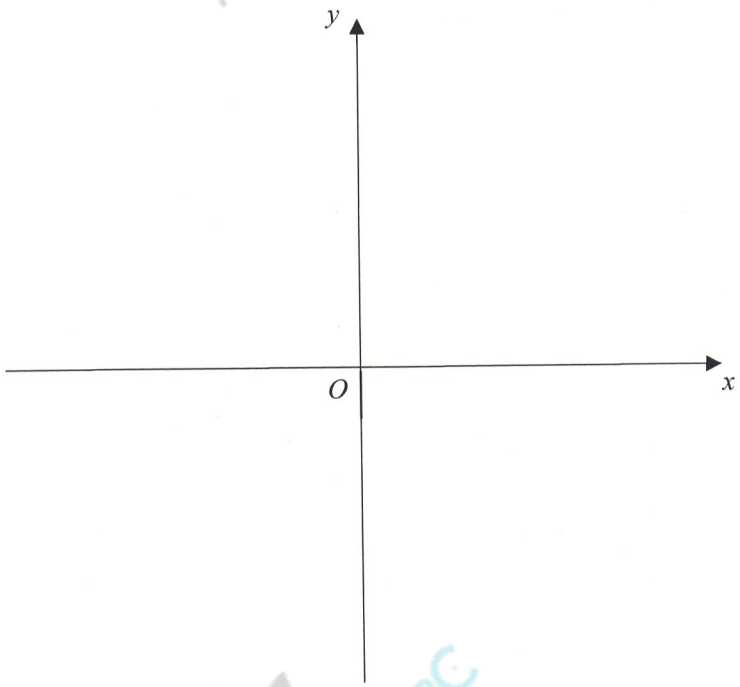
Answer ..... [1]

(b) Write down the equation of the line of symmetry of the graph $y = x^2 - 2x + 9$.

Answer ..... [1]

(c) Sketch the graph of $y = x^2 - 2x + 9$. Indicate the values where the graph crosses the x and y-axes and the turning point of the curve.

Answer



[2]

10 **(a)** Simplify the following expressions, leaving your answers in positive index.

(i) $\dfrac{15xy^2}{\left(3xy^{-2}\right)^3}$,

Answer ..... [2]

(ii) $\left(\dfrac{-3w^2}{2y}\right)^{-4}$.

Answer ..... [2]

(b) Solve $\dfrac{5^x}{\sqrt[3]{5}} = 5^{-x}$.

Answer $x =$ ..... [2]

11 **(a)** Express 2520 as a product of its prime factors.

Answer ..... [1]

(b) The number $\frac{2520}{p}$ is a perfect square.

Find the smallest possible integer value of p.

Answer $p =$ ..... [1]

(c) q is a number between 800 and 1000.
The highest common factor of q and 2520 is 84.
Find the smallest possible value of q.

Answer $q =$ ..... [2]

12 Roy has a map drawn to the scale 1 : 200 000.

(a) The distance on the map between Gardens by the Bay and Night Safari is 14.1 cm.
Calculate the actual distance, in kilometres, between Gardens by the Bay and Night Safari.

Answer ..... km [1]

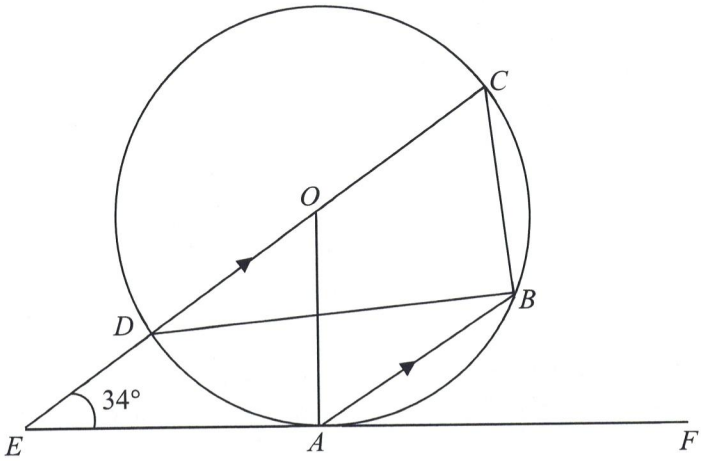
(b) The area of Gardens by the Bay is 1.01 km^2.
Find the area of Gardens by the Bay, in square centimetres, represented on Roy's map.

Answer ..... cm^2 [2]

(c) The area covered by Gardens by the Bay on Tim's map is 4 times that on Roy's map.
Find the scale of Tim's map, giving your answer in the form 1 : n.

Answer 1 : ..... [2]

13 A, B, C and D are points on the circumference of a circle with centre O.
EAF is tangent to the circle at A and meets COD produced at E.
AB is parallel to DC and angle $OEA = 34°$.



(a) Find angle AOE. Give a reason for each step of your working.

Answer Angle AOE = ... [1]

(b) Find angle ABD. Give a reason for each step of your working.

Answer Angle ABD = ... [1]

(c) Find angle ADC. Give a reason for each step of your working.

Answer Angle ADC = ... [2]

14 The equation of the line l_1 is $x+3y-10=0$. It crosses the x-axis at point A.

(a) Find the coordinates of point A.

Answer (..... , ...) [1]

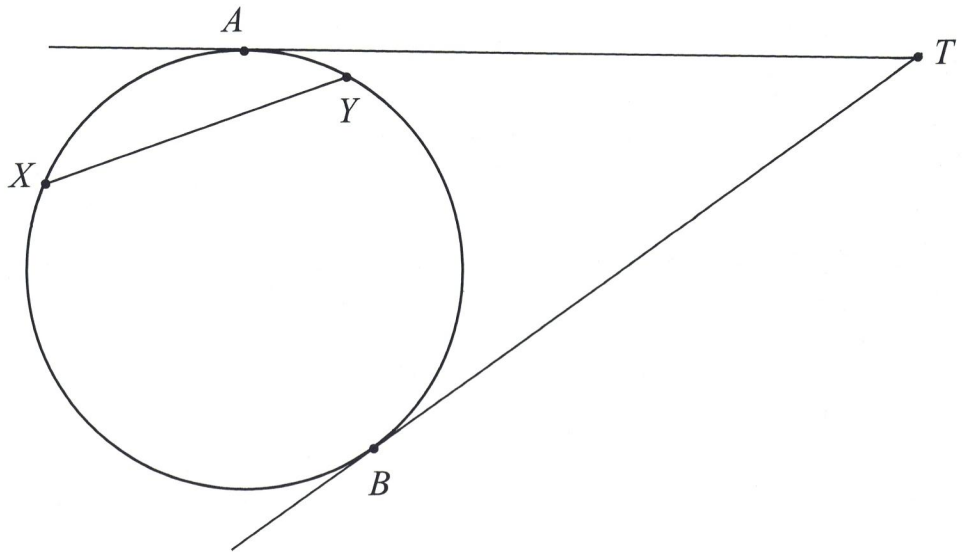
(b) A second line l_2 passes through the point $(0, -3)$ and is parallel to l_1.
Find the equation of l_2.

Answer ..... [2]

(c) The coordinates of a point B is $(k, 4)$, where $k < 0$.
Given that the length of the line segment AB is $\sqrt{137}$ units, find the value of k.

Answer $k =$ ..... [3]

15 In the diagram below, TA and TB are tangents to a circle at A and B respectively. XY is a chord on the circle.



(a) Construct the perpendicular bisector of XY. [1]

(b) Construct the angle bisector of angle ATB. [1]

(c) The two bisectors constructed in **(a)** and **(b)** intersect at point P.
Label the point P.
Hence, state how triangle PAT is related to triangle PBT.
Justify your answer with clear reasons.

Answer Triangles PAT and PBT are .....

.....

.....

.....

.....

..... [3]

END OF PAPER

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ANG MO KIO SECONDARY SCHOOL
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Paper 2

4052/02
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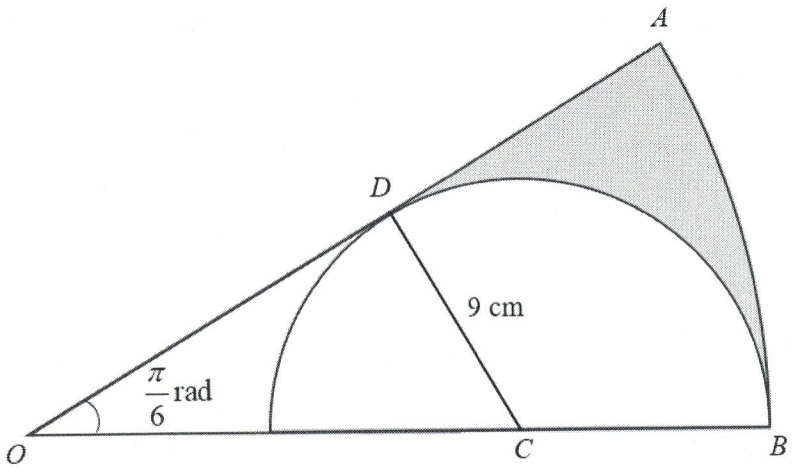
1 **(a)** Simplify $\dfrac{4p^2-(p+q)^2}{q-p}$.

Answer ..... [3]

(b) Rearrange the formula $m=\dfrac{3+p^2}{p^2-m}$ to make p the subject of the formula.

Answer $p=$ ..... [2]

2 The diagram shows a sector OAB of a circle with centre O and radius OA. A semi-circle with centre C and radius 9 cm is drawn inside the sector such that OA is the tangent to the semicircle at D. $\angle AOB = \frac{\pi}{6}$ radians and $CD = 9$ cm.



(a) Show that the length of $OC = 18$ cm.

Answer

[1]

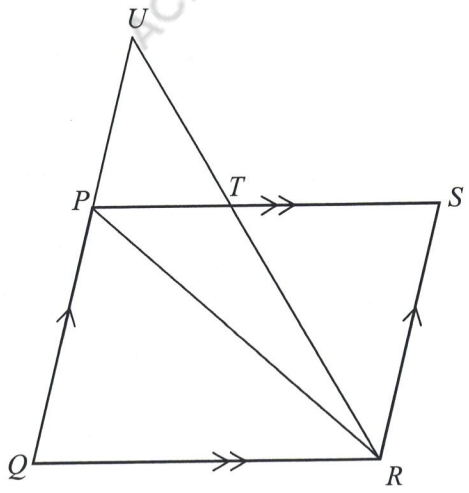
(b) Find the perimeter of the shaded region.

Answer ..... cm [3]

(c) Find the area of the shaded region.

Answer ..... cm^2 [3]

3 The diagram shows a parallelogram $PQRS$ with QP produced to U.
T is the point of intersection of PS and RU.



(a) Show that triangle PTU and triangle STR are similar.
Give a reason for each statement you make.

.....

.....

.....

..... [2]

(b) The ratio of $RT : RU = 2 : 3$.

(i) State the ratio of $PT : TS$.

Answer ..... : ..... [1]

(ii) Given that the area of triangle PTU is 20 cm^2, find the area of triangle PRU.

Answer ..... cm^2 [2]

4 Two geometrically similar cylinders are made of the same material.
The ratio of the surface areas of these two cylinders is 16 : 49.
The smaller cylinder has a volume of 87.8 cm^3, and a mass of 2.09 kg.
The base area of the larger cylinder is 117.6 cm^2.

(a) Find the height of the smaller cylinder.

Answer ..... cm [2]

(b) Find the mass of the larger cylinder.

Answer ..... kg [3]

5 The variables x and y are connected by the equation $y = \frac{2}{x} - \frac{x^2}{4} + 5$.

Some corresponding values of x and y are given in the table below.

x	–6	–5	–4	–3	–2	–1.5	–1	–0.5	–0.3
y	–4.3	–1.7	p	2.1	3	3.1	2.8	0.9	–1.7

(a) Find the value of p.

Answer $p =$ ..... [1]

(b) On the grid opposite, draw the graph of $y = \frac{2}{x} - \frac{x^2}{4} + 5$ for $-6 \leq x \leq 0$. [3]

(c) Use your graph to solve the equation $\frac{2}{x} - \frac{x^2}{4} + 5 = 1$.

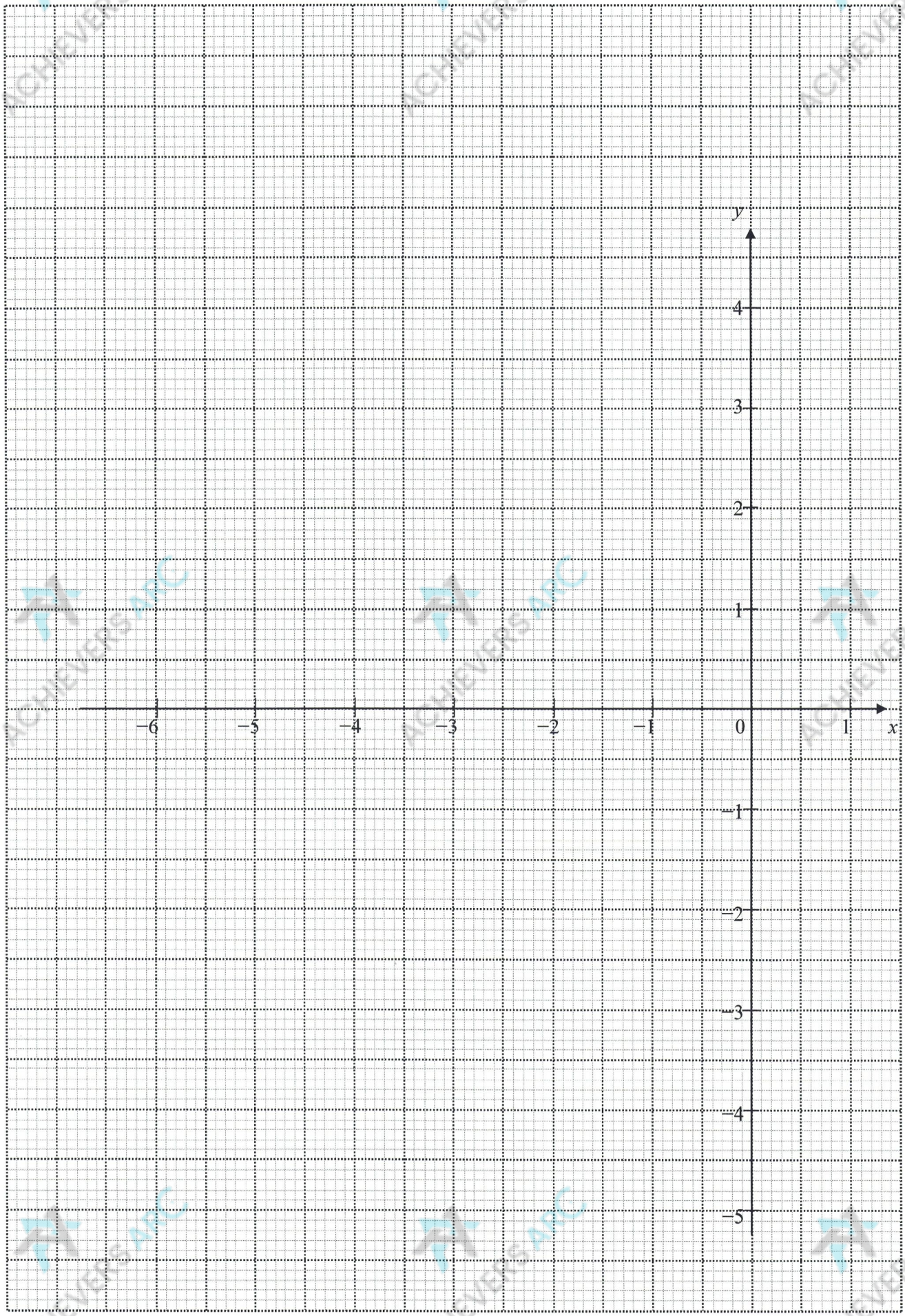
Answer $x =$ ..... , ... [2]

(d) By drawing a tangent, find the gradient of the curve at the point $x = -1$.

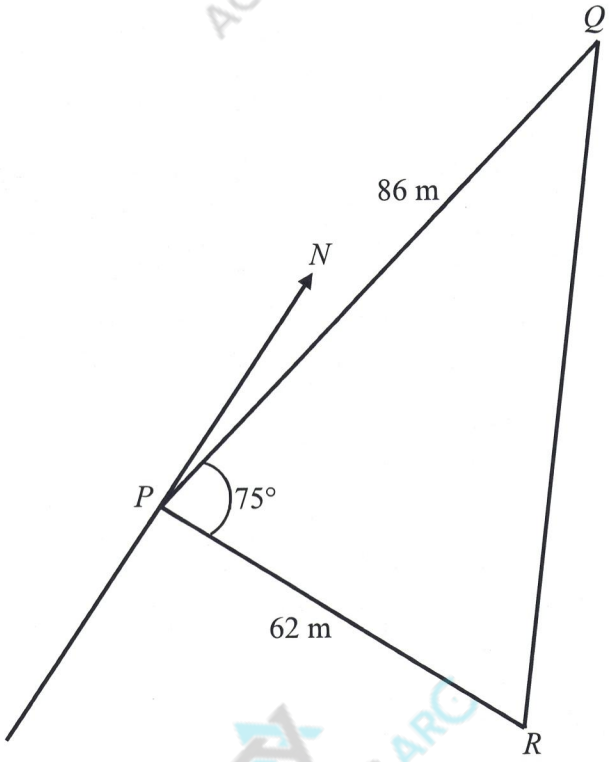
Answer ..... [2]

(e) By drawing a suitable straight line on the same axes, solve $x^3 + 2x^2 - 24x - 8 = 0$.

Answer $x =$ ..... , ... [3]


y
x
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−5
−4
−3
−2
−1
1

6 In the diagram, P, Q and R are three points on a horizontal field.
R is due east of P, $PR = 62$ m, $PQ = 86$ m and and $\angle RPQ = 75°$.



(a) Calculate

(i) the bearing of P from Q,

Answer ..... ° [1]

(ii) the distance QR,

Answer ..... m [3]

(iii) $\angle PQR$.

Answer ..... ° [2]

(b) A helicopter is hovering directly above P.
The angle of elevation of the helicopter from Q is 24°.
Calculate the height of the helicopter from the ground.

Answer ..... m [2]

(c) A target moves along RQ. A soldier in the helicopter shoots the target when the angle of depression of the target from the helicopter is at its greatest.
Calculate the angle of depression.

Answer ..... ° [3]

7 A water tank has a capacity of 900 litres. A pump can fill the tank completely with water at a constant rate of x litres per minute, while a tap can fill the tank completely at a constant rate of $(x-2)$ litres per minute.

Expressing your answers in hours,

(a) (i) find the time taken for the pump to fill up the tank completely with water,

Answer ..... h [1]

(ii) find the time taken for the tap to fill up the tank completely with water.

Answer ..... h [1]

(b) The pump takes half an hour faster than the tap to fill up the tank completely.
Form an equation, in terms of x, to represent this information and show that it reduces to

$$x^2 - 2x - 60 = 0 .$$ [2]

(c) Solve the equation $x^2 - 2x - 60 = 0$.

Give your answer correct to 1 decimal place.

Answer $x =$ ..... or ..... [3]

(d) Hence, find the time taken to fill up the tank if the pump and the tap were turned on at the same time.

Give your answer correct to the nearest minute.

Answer ..... minutes [2]

8 Figure I shows a sector OAB of a circle with centre O and radius 12 cm.
The region $CABD$ is cut out from the sector and folded to form the frustum shown in Figure II.
The small cone in Figure II has a radius of 2 cm.

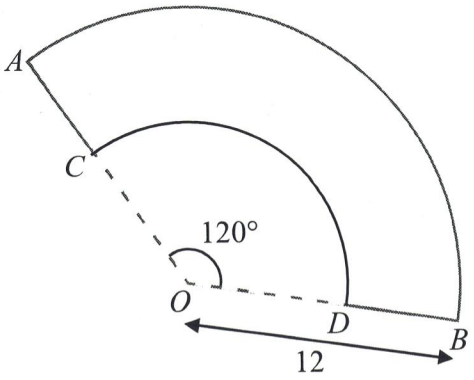


Figure I

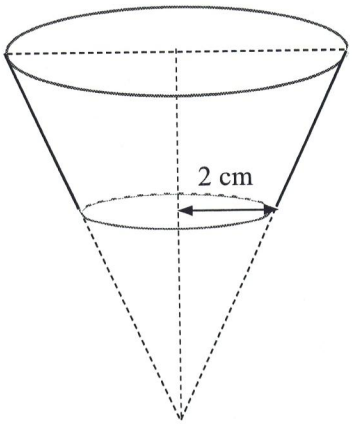


Figure II

(a) Show that the radius of the top of the frustum is 4 cm. [2]

(b) Find the volume of the frustum.

Answer ..... cm^3 [2]

(b) A cylinder of height 7 cm is attached to the frustrum to form a vase as shown in Figure III. Water is poured into the vase at a constant rate, and it takes 10 seconds to fill up the vase.

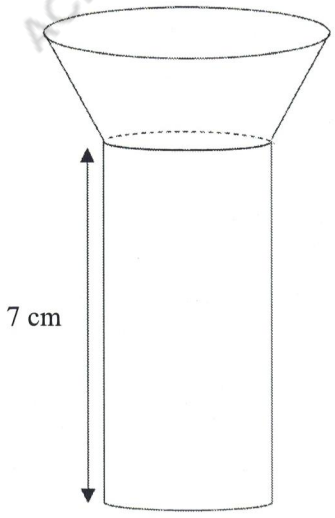


Figure III

The manufacturer wants to modify the vase such that the height of the cylinder is 1.5 times of its current height. The manufacturer claims that it will take approximately 2.5 seconds more to fill up the modified vase.

Assuming that the water is being filled up at the same rate as before, is the manufacturer's claim correct?

Justify your answers with calculations. [3]

END OF PAPER

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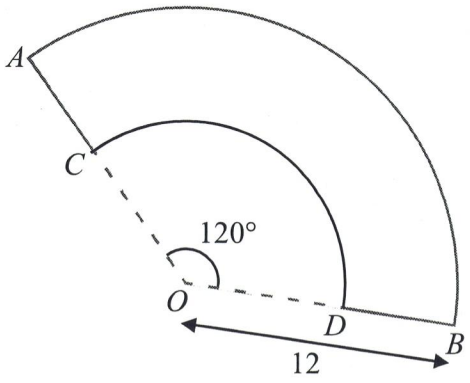

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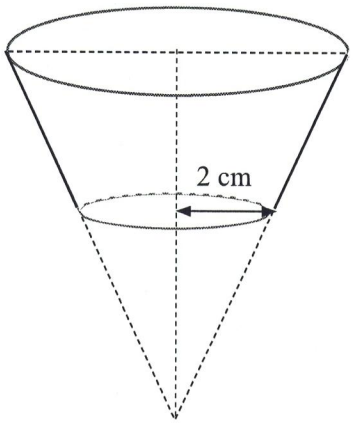

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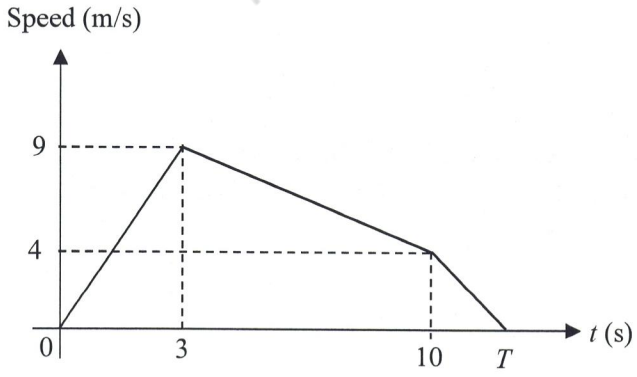

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Answer ..... m/s [2]

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Answer $T =$ ..... [2]

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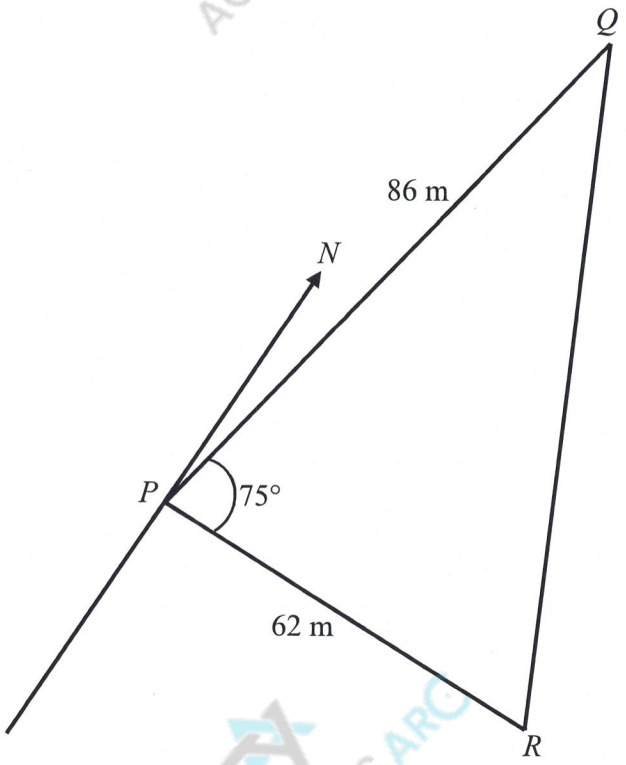

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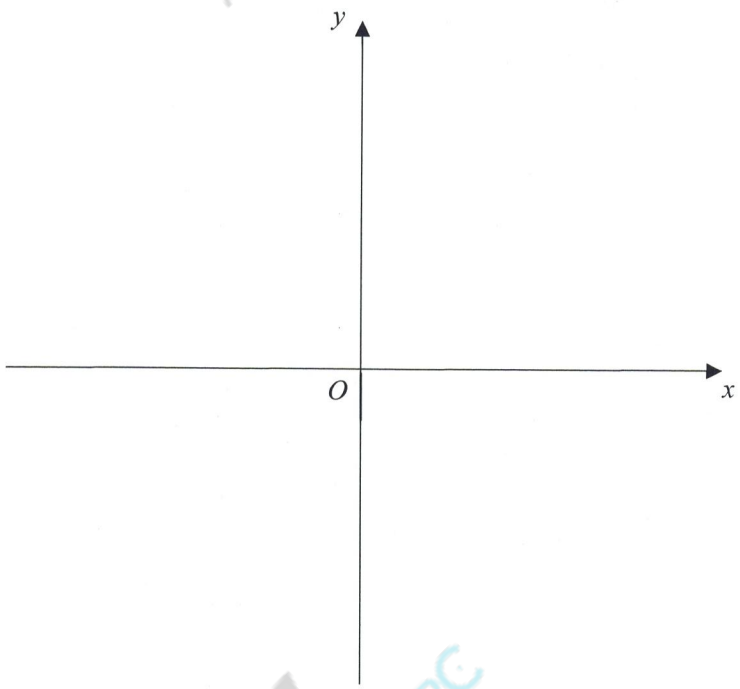

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Answer $p =$ ..... [1]

(b) On the grid opposite, draw the graph of $y = \frac{2}{x} - \frac{x^2}{4} + 5$ for $-6 \leq x \leq 0$. [3]

(c) Use your graph to solve the equation $\frac{2}{x} - \frac{x^2}{4} + 5 = 1$.

Answer $x =$ ..... , ... [2]

(d) By drawing a tangent, find the gradient of the curve at the point $x = -1$.

Answer ..... [2]

(e) By drawing a suitable straight line on the same axes, solve $x^3 + 2x^2 - 24x - 8 = 0$.

Answer $x =$ ..... , ... [3]

11 **(a)** Express 2520 as a product of its prime factors.

Answer ..... [1]

(b) The number $\frac{2520}{p}$ is a perfect square.

Find the smallest possible integer value of p.

Answer $p =$ ..... [1]

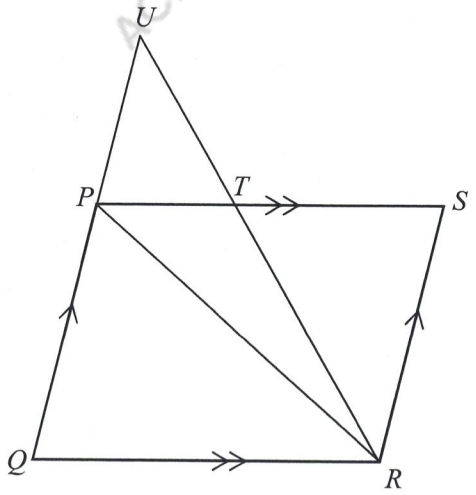
(c) q is a number between 800 and 1000.

The highest common factor of q and 2520 is 84.

Find the smallest possible value of q.

Answer $q =$ ..... [2]

3 The diagram shows a parallelogram $PQRS$ with QP produced to U.
T is the point of intersection of PS and RU.



(a) Show that triangle PTU and triangle STR are similar.
Give a reason for each statement you make.

.....

.....

.....

..... [2]

(b) The ratio of $RT : RU = 2 : 3$.

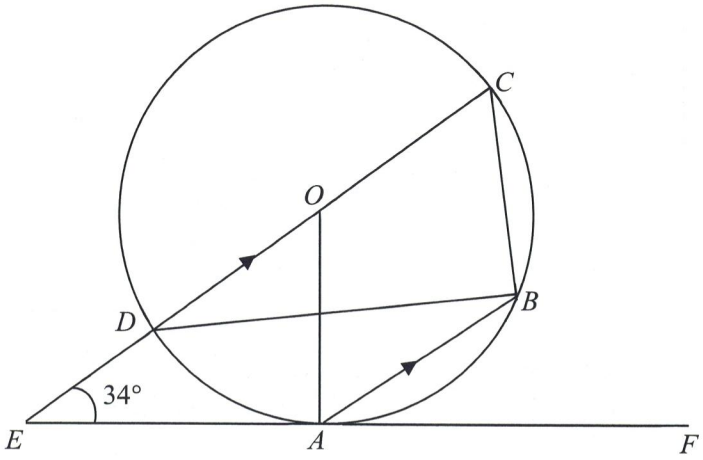
(i) State the ratio of $PT : TS$.

Answer ..... : ..... [1]

(ii) Given that the area of triangle PTU is 20 cm^2, find the area of triangle PRU.

Answer ..... cm^2 [2]

13 A, B, C and D are points on the circumference of a circle with centre O.
EAF is tangent to the circle at A and meets COD produced at E.
AB is parallel to DC and angle $OEA = 34°$.



(a) Find angle AOE. Give a reason for each step of your working.

Answer Angle AOE = ... [1]

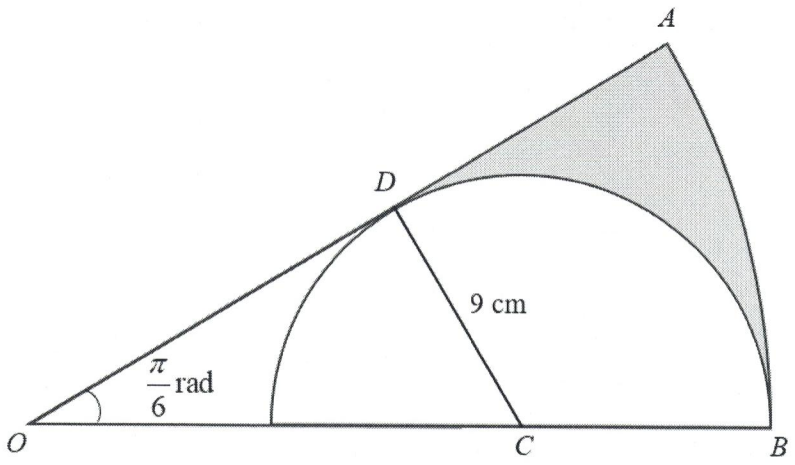
(b) Find angle ABD. Give a reason for each step of your working.

Answer Angle ABD = ... [1]

(c) Find angle ADC. Give a reason for each step of your working.

Answer Angle ADC = ... [2]

2 The diagram shows a sector OAB of a circle with centre O and radius OA. A semi-circle with centre C and radius 9 cm is drawn inside the sector such that OA is the tangent to the semicircle at D. $\angle AOB = \frac{\pi}{6}$ radians and $CD = 9$ cm.



(a) Show that the length of $OC = 18$ cm.

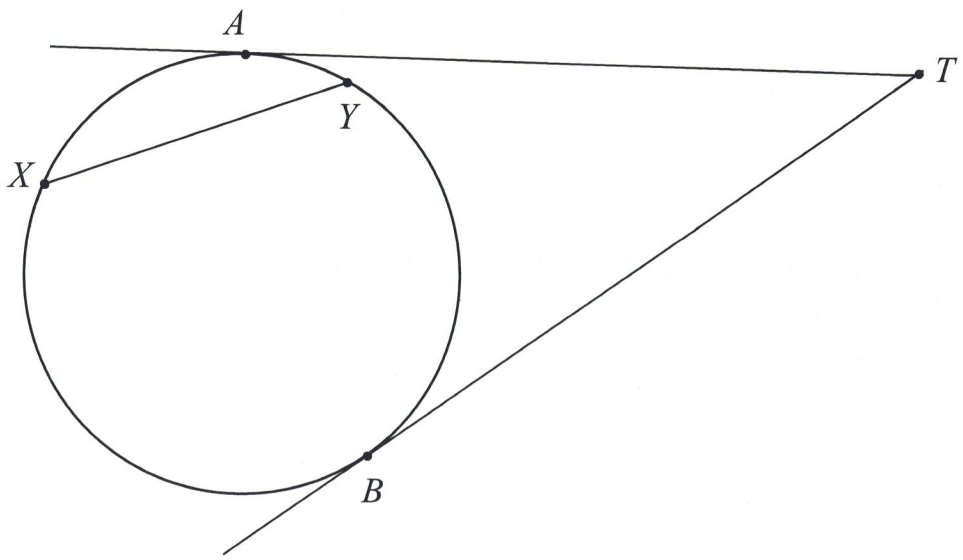
Answer

[1]

(b) Find the perimeter of the shaded region.

Answer ..... cm [3]

15 In the diagram below, TA and TB are tangents to a circle at A and B respectively. XY is a chord on the circle.



(a) Construct the perpendicular bisector of XY. [1]

(b) Construct the angle bisector of angle ATB. [1]

(c) The two bisectors constructed in **(a)** and **(b)** intersect at point P.
Label the point P.
Hence, state how triangle PAT is related to triangle PBT.
Justify your answer with clear reasons.

Answer Triangles PAT and PBT are .....

.....

.....

.....

.....

..... [3]

END OF PAPER

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curve surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector Area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$