BUKIT VIEW SECONDARY SCHOOL

Secondary Three Express
End-of-Year Examination 2024

CANDIDATE NAME

CLASS

REGISTER NUMBER

Additional Mathematics

4049
3 October 2024
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided on top of this cover page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

Marks
90

Setter: Mrs Kelly Chew-Au

Parents' Signature:

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

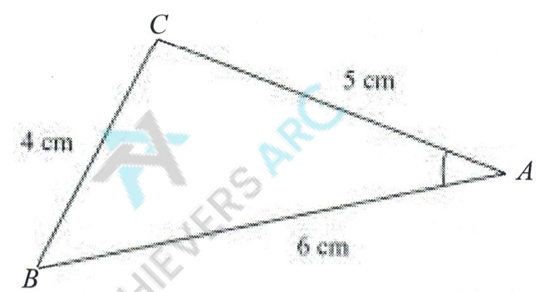
$$\Delta = \frac{1}{2}bc \sin A$$

1. **(a)** Given that $\cos\theta = \frac{2}{3}$ and $\sin\theta < 0$, find the values of

(i) $\tan\theta$ [2]

(ii) $\sec(90° - \theta)$ [1]

(b) Triangle ABC with AB = 6 cm, BC = 4 cm and AC = 5 cm is shown in the diagram below.



(i) Show that $\cos A = \frac{3}{4}$. [2]

(ii) Hence or otherwise, find the **exact** value of sin A in the simplest form. [3]

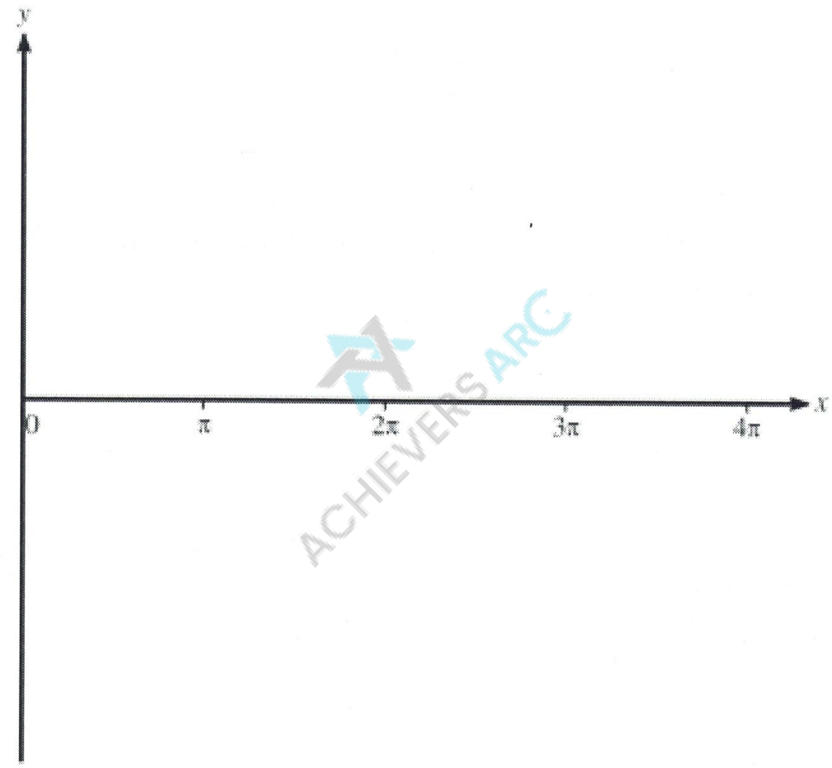
2. **(a)** Solve $\sqrt{x-2}+2=\sqrt{3x-2}$. [4]

(b) A cylinder has a radius of $\left(\frac{1}{\sqrt{2}-1}\right)$ cm and a height of $\left(\sqrt{2}+1\right)$ cm. Find the volume of the cylinder in exact form, leaving your answer in terms of π. [5]

3. A curve has equation $y = 2 + 3\sin\frac{1}{2}x$ for $0 \le x \le 4\pi$.

(a) State greatest and least values of y. [2]

(b) Sketch the curve $y = 2 + 3\sin\frac{1}{2}x$ for $0 \le x \le 4\pi$. [3]



(c) Find the number of solutions of the equation $3\sin\frac{1}{2}x = 4 - \frac{2x}{\pi}$. [2]

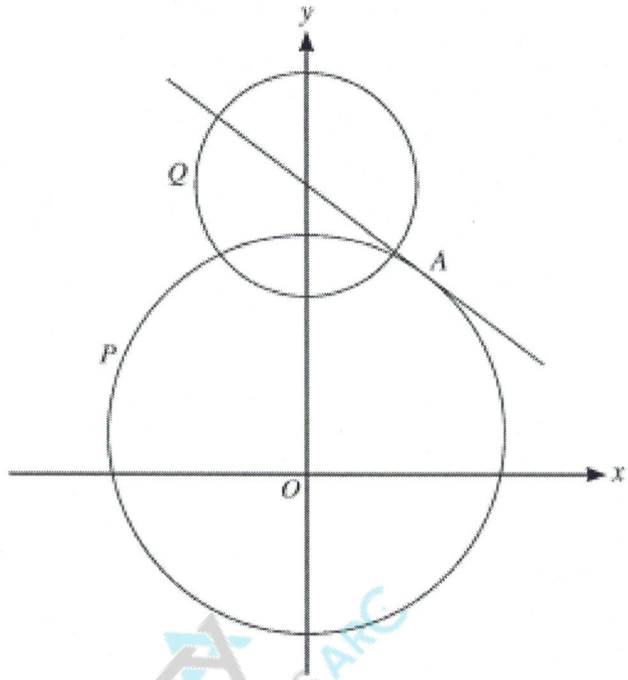
4. The polynomial p(x) is defined by $p(x) = x^3 + x^2 + kx - 12$, where k is a constant.
It is given that $(x - 3)$ is a factor of p(x).

(a) Show that $k = -8$. [2]

(b) By showing all necessary working clearly, determine the number of real roots for $p(x) = 0$. [4]

(c) Hence or otherwise, express $\dfrac{2x^3+3x^2-4x-19}{x^3+x^2+kx-12}$ as partial fractions. [6]

5. The diagram shows a circle P with centre (0, 2) and radius 10 and the tangent to the circle at the point A with coordinates (6, 10). It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$.

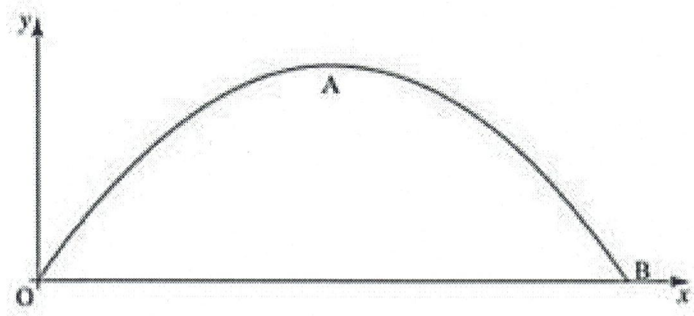


(a) Find the equation of the tangent to the circle P at A. [3]

(b) Find the equation of circle Q in the general form. [3]

(c) Find the y-coordinates of the points of intersection of the tangent and circle Q, giving the answers in surd form. [4]

6. The diagram shows the arch of a bridge where OB is the horizontal road level. The bridge can be modelled by the equation $y = 2.5x - 0.3125x^2$. All measurements are in metres.

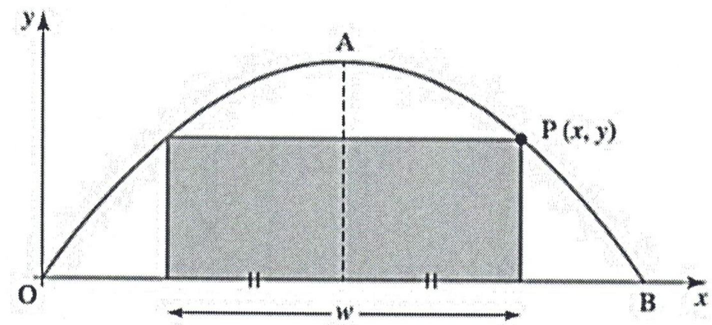


(a) Calculate the length of OB. [2]

(b) Calculate the height above the road at point A, the highest point of the arch. [2]

A car towing a caravan needs to drive under the bridge. The caravan is 5 metres wide and has a height of 2 metres. Only one single lane of traffic can pass under the bridge. To avoid accidents, the bridge engineers decided to place height and width limits. Only vehicles whose height and width fit into the greatest allowable dimensions are permitted to travel under the bridge.

Point P(x, y) lies on the curve and is the corner of a rectangle formed by the new height and width limits.

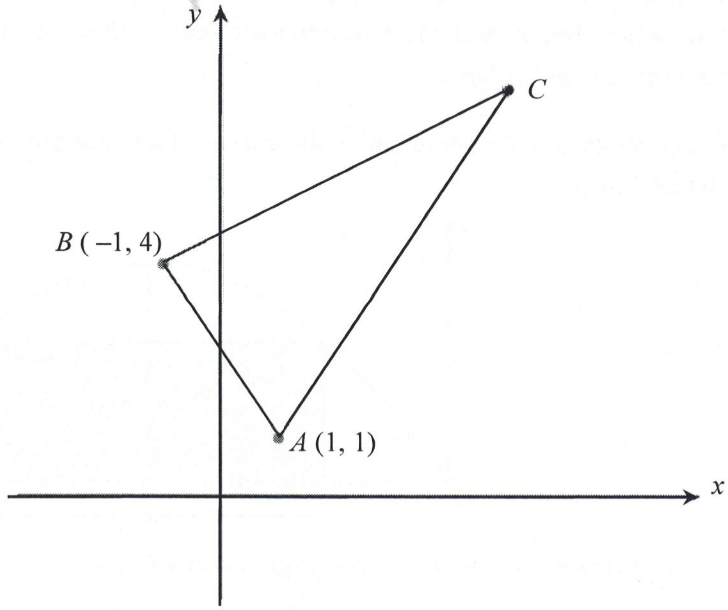


(c) Express the width of the rectangle in terms of x. [2]

(d) If the height limit is 3.2 metres, calculate the x-coordinate of P. [3]

(e) Determine whether the caravan would be permitted to be towed under the bridge with the new limits. [2]

7. The diagram shows a triangle ABC. The gradients of the lines AB, AC and BC are $-3m$, $3m$ and m respectively.



(a) Find the value of m. [2]

(b) Find the coordinates of point C. [4]

(c) The perpendicular bisector of BC meets the x axis at point D. Find the coordinates of point D. [4]

(d) Hence, find the area of quadrilateral $ABCD$. [2]

8. **(a)** Solve the following equations.

(i) $1 - \cos x = 5\sin^2 x$ for $0° \leq x \leq 360°$. [5]

(ii) $4\sin x\cos x = \sqrt{3}$ for $0 \leq x \leq 2\pi$, leaving the answer in exact form. [4]

(b) Prove the identity $\dfrac{\sin x}{\cos x+\sin x}+\dfrac{1-\cos x}{\cos x-\sin x}=\dfrac{\cos x+\sin x-1}{1-2\sin^2 x}$. [3]

9. **(a)** Solve $x^2 > 121$. [2]

(b) A curve has the equation $y = 2x^2 + 4(p + 2)x + 8p + q + 8$, where p and q are constants. The curve meets the y-axis at $(0, 18)$.
Given further that the curve has **no** x intercepts, show that $-5 < p < 1$. [7]

END OF PAPER