Class	Index Number	Name

BUKIT MERAH SECONDARY SCHOOL

END-OF-YEAR EXAMINATION 2024
SECONDARY 3 EXPRESS

ADDITIONAL MATHEMATICS **4049**

8 October 2024
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Calculator Model:

For Examiner's Use

This document consists of **23** printed pages and **1** blank page.

Setter: Ms Jasmine Ng

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Solve the equation $4^x - 2^{x+2} = 32$. [4]

2 Express $\dfrac{x+14}{(x-2)(x^2+4)}$ in partial fractions. [5]

3 A cuboid has a square base. The length of each side of the base is $\left(\sqrt{3}-\sqrt{2}\right)$ m and the volume of the cuboid is $\left(5\sqrt{2}+4\sqrt{3}\right)$ m^3. Find, **without using a calculator**, the height of the cuboid, in m, in the form of $\left(a\sqrt{2}+b\sqrt{3}\right)$, where a and b are integers. [5]

4 Given that $\sin A = -\frac{7}{25}$, where $\frac{\pi}{2} < A < \frac{3\pi}{2}$.

Find, **without using a calculator**, the exact value of

(a) $\operatorname{cosec} A$, [1]

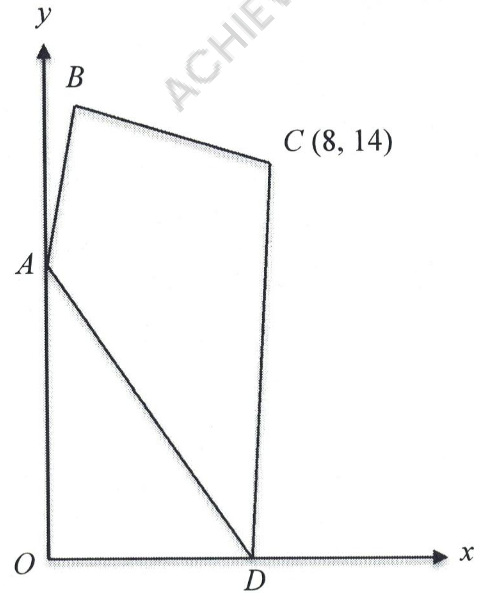
(b) $\tan A$, [2]

(c) $\cos(-A)$. [1]

(d) $\cos^2 \frac{\pi}{6} - \sin(\pi - A)$ [2]

5 Find the set of values of m for which the curve $y = mx^2 - 4x + 36m$ lies completely above the x-axis. [4]

6



The diagram shows a quadrilateral *ABCD*. The point *A* lies on the *y*-axis where $y = 10$.
The point *D* lies on the *x*-axis and the point *C* is (8, 14).

(a) Given that line *AD* is parallel to $5x + 3y - 10 = 0$, find the coordinates of *D*. [3]

(b) Show that the point D does not lie on the perpendicular bisector of AC. [4]

(c) The point $B(a,b)$ is such that the length of AB is $\sqrt{29}$ units.

(i) Show that $a^2+b^2-20b+71=0$. [2]

(ii) The area of $ABCD$ is 68 units^2. Determine the coordinates of B, explaining why the diagram is necessary. [4]

7 An experiment was carried out in the laboratory to test out the growth of a favourable bacteria. The experiment started at 12 pm on a particular day. It is given that P is the number of bacteria present t hours after the start of the experiment and it can be modelled by the equation $P = Ae^{kt}$, where A and k are constants.

There were 5000 bacteria at the start of the experiment.

(a) Find the value of A. [1]

(b) The number of bacteria would triple every 3.5 hours. Find the value of k. [3]

(c) Find the number of bacteria at 6.30 pm on the same day. [1]

8 The term independent of x in the expansion of $\left(\frac{x}{2}-\frac{h}{x^2}\right)^9$ is $-\frac{2625}{2}$.

(a) Find the value of h. [4]

(b) Using the value of h found in part (a), find the term independent of x in the expansion of $(2+x^3)\left(\frac{x}{2}-\frac{h}{x^2}\right)^9$. [4]

9 The function $f(x) = x^3 + ax^2 + bx - 3$, where a and b are constants, is divisible by $x - 3$ and leaves a remainder of 15 when divided by $x + 2$.

(a) Find the value of a and of b. [4]

(b) Factorise $f(x)$ as a product of a linear factor and a quadratic factor. [2]

(c) Determine, showing all necessary working, the number of real roots of the equation $f(x) = 0$. [2]

10 A circle, with centre C, has the equation $x^2 + y^2 - 6x + 12y - 244 = 0$.

(a) Find the coordinates of C and the radius of the circle. [3]

(b) Find the equation of the tangent to the circle at $(-12, 2)$. [3]

(c) Find the values of k if the circle touches the line $y = k$. [2]

(d) Determine whether the point (15, 5) lies inside, outside or on the circle. [2]

11 (a) The graph of $y = 4\sin ax + b$ has its maximum point at (45°, 7) and the minimum point after this is at (135°, −1). Show that $a = 2$ and find the value of b. [2]

(b) Sketch the graph of $y = 4\sin ax + b$ for $0° \leq x \leq 360°$. [2]

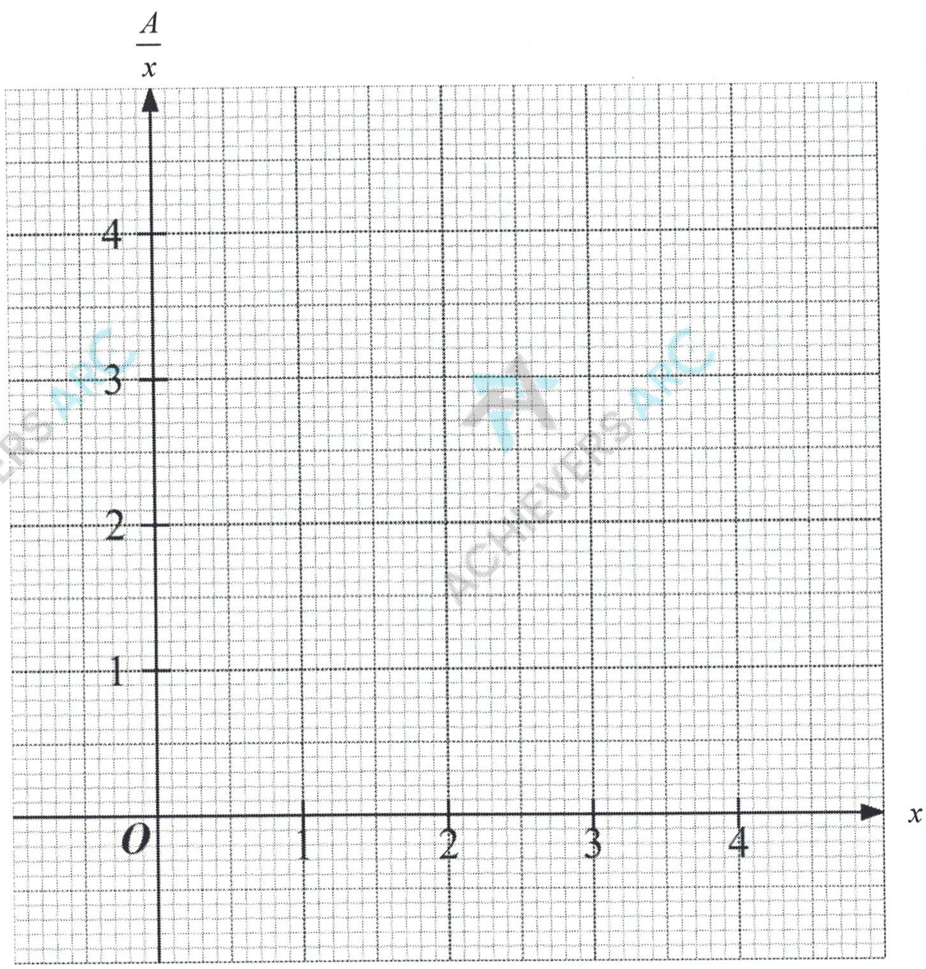
12 (a) Show that $\log_2\left(\frac{x}{4}\right)+\log_2 32-\log_2 4=\log_2 2x$. [1]

(b) Hence solve the equation $\log_2\left(\frac{x}{4}\right)+\log_2 32-\log_2 4=3+\log_{\sqrt{2}}(x-1)$, giving your answer in the form of $\frac{a+\sqrt{b}}{c}$, where a, b and c are integers. [5]

13 A trapezium of area, A cm^2, has parallel lines of length px cm and q cm and its perpendicular height is x cm, where p and q are constants. Measurements of x and A are shown in the table below.

x	1	2	3	4
A	1.75	5	9.75	16

(a) On the grid below plot $\frac{A}{x}$ against x and draw a straight line graph. [2]

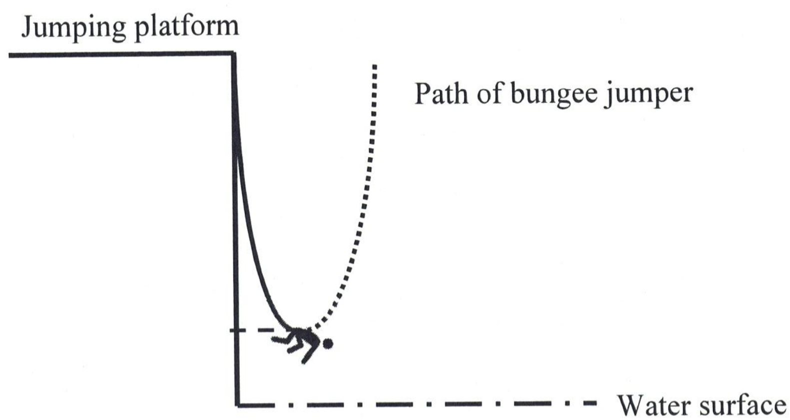


(b) Use your graph to estimate

(i) the values of the constants p and q, [3]

(ii) the area of the trapezium when the perpendicular height is 3.5 cm. [2]

14 The path of a bungee jumper from the Adrenaline Bungee Jumping site can be modelled by a quadratic function with its graph as shown below.



The vertical height, h m, from the water surface is given by $h = 50x^2 - 100x + k$, where x m is the horizontal distance from the jumping platform and k is a constant.

(a) Given that a bungee jumper is at his minimum height of 8 m above the water surface, find the value of k. [3]

(b) Ben's current personal record for bungee jumping is 52 m, the distance measured from the jumping platform to the minimum height. Explain whether Ben will break his own personal record if he jumps at the Adrenaline Jumping site. [2]

End of Paper

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