8**

Name	Class				Index Number		

BROADRICK SECONDARY SCHOOL
SECONDARY 3 G3
END-OF-YEAR EXAMINATION 2024

ADDITIONAL MATHEMATICS 4049

Candidates answer on the Question Paper.
No Additional Materials are required.

October 2024

2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
Write the question number attempted in the left column in the box provided.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use		
Error in	Question Number	Marks Deducted
No/Wrong Units		
Rounding-off		
Reasoning		
Presentation		

For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total Marks	/90

This document consists of **19** printed pages.

Setter : Yong JJ

[Turn Over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The variables x and y are related in such a way that when a graph of $\frac{1}{y}$ is plotted against $\frac{1}{\sqrt{x}}$, a straight line which passes through (1, 2) and (5, 4) is obtained.
Find the value of y when $x = 9$. [4]

2 **(a)** Given that $\sin\theta = k$, where θ is an acute angle, express $\sin\theta + \tan\theta$ in terms of k. [2]

(b) Without using a calculator, find the exact value of $\frac{2\sin 60}{\cos 45 + \tan 45}$. [3]

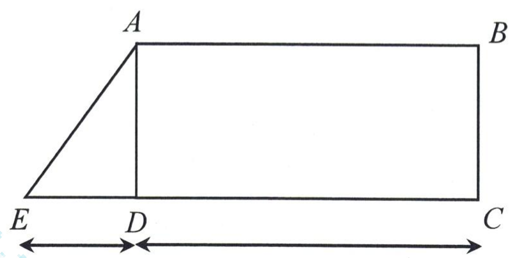
Give your answer in the form $a\sqrt{b} - \sqrt{c}$, where a, b and c are integers.

3 **(a)** Sketch the graph of $y = 2\cos x$ and $y = -\sin\left(\frac{x}{2}\right)$ on the same axes for $0° \leq x \leq 360°$. [4]

(b) Find the number of solutions for $2\cos x - \sin\left(\frac{x}{2}\right) = 0$ for $0° \le x \le 360°$. [1]

---

4 The diagram below shows a trapezium *ABCDE* and a rectangle *ABCD*.
It is given that $BC = \left(\sqrt{18} + 2\right)$ cm, $ED = \left(\sqrt{18} - 2\right)$ cm and $DC = \left(\sqrt{18} + 4\right)$ cm.



(a) Find the area of the trapezium, giving your answer in the form $a + b\sqrt{c}$, where *a*, *b* and *c* are integers and *c* is smaller than *b*. [3]

(b) Find tan $\angle AED$, giving your answer in the form $\dfrac{d+e\sqrt{2}}{f}$, where d, e and f are integers. [3]

5 **(a)** Find the remainder when $4x^3+12x^2+9x+2$ is divided by $x+3$. [1]

(b) Solve $4x^3 + 12x^2 + 9x + 2 = 0$. [5]

(c) Explain why the equation $4(8^x) + 12(4^x) + 9(2^x) + 2 = 0$ has no solutions. [2]

6 **(a)** Explain why there is no constant term in the expansion of $\left(x^2+\frac{3}{x}\right)^7$. [3]

(b) **(i)** Find the first 4 terms in the expansion of $\left(1+\frac{x}{4}\right)^{10}$ in ascending powers of x. [2]

(ii) Find the coefficient of x^3 in the expansion of

$$2\left(1+\frac{x}{4}\right)^{10}+3\left(1+\frac{x}{4}\right)^{11}+4\left(1+\frac{x}{4}\right)^{12}.$$ [3]

7 **(a)** Determine the set of values of k for which the equation $x^2 - 4x - 1 = 2kx - 10k$ has no real roots. [4]

(b) Hence, state, giving a reason, what can be deduced about the curve $y = (x-2)^2 - 5$ and the line $y = 6x - 30$. [2]

8 A ball is thrown from a cliff overlooking the sea. The vertical height of the ball from the sea level, h metres is given by $h = -t^2 + 14t + 24$, where t is the time in seconds after the ball is thrown.

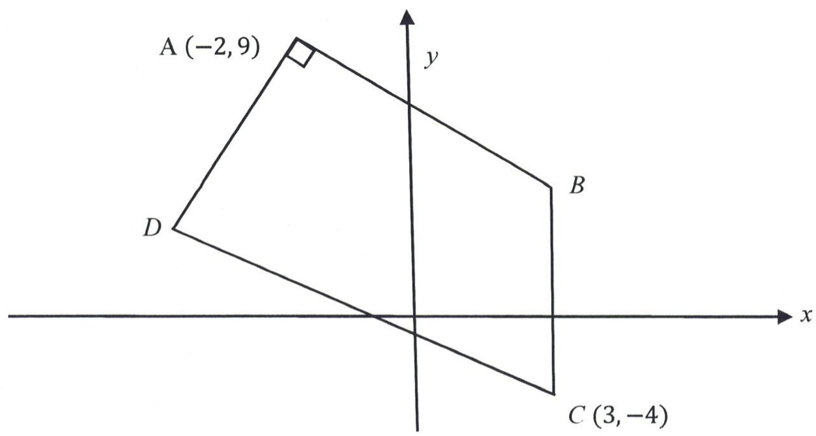
(a) Write down the height of the cliff. [1]

(b) Find the vertical height of the ball from sea level after 3 seconds. [1]

(c) Explain why the ball can never reach a height of 74 metres above sea level. [3]

(d) Find the duration, in seconds, when the ball is at least 60m above sea level. Give your answer correct to 2 decimal places. [3]

9 $ABCD$ is a quadrilateral where the coordinates of A and C are $(-2, 9)$ and $(3, -4)$ respectively. Point B is 9 units vertically above point C and $\angle BAD = 90^\circ$.



(a) Write down the coordinates of point B. [1]

(b) Show that the equation of line AD can be written as $4y = 5x + 46$. [2]

(c) Given that the length of CD is $\sqrt{\frac{605}{4}}$, show that

$$(x-3)^2+\left(\frac{5x+46}{4}+4\right)^2=\frac{605}{4}$$

[2]

(d) Solve $(x-3)^2+\left(\frac{5x+46}{4}+4\right)^2=\frac{605}{4}$ and find the coordinates of point D, given that the x-coordinate of point D is an integer.

[4]

(e) Hence, find the area of the quadrilateral *ABCD*. [2]

---

10 The table below shows some experimental readings of two variables, x and y.

x	1	1.2	1.4	1.6	1.8
y	1	1.99	3.42	5.28	7.85

It is known that x and y are related by the equation $y = kx^3 + hx$, where k and h are constants.

(a) Using a suitable scale for your axes, plot the graph of $\frac{y}{x}$ against x^2 on the graph paper on the next page. [3]

(b) Use your graph drawn in **(a)** to estimate the value of k and h. [4]

11 **(a)** Find the centre and radius of the circle with equation $x^2 + y^2 + 6x - 14y + 57 = 0$. [3]

(b) Explain why a straight line with equation $x - 2y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 20$. [3]

(c) A circle, C_1 has the equation $x^2 - 8x + y^2 - 16y = -74$.

A larger circle, C_2, has the equation $(x-1)^2 + (y-8)^2 = 36$

Determine if C_1 lies completely within C_2. [5]

12 **(a)** Solve the simultaneous equations

$$2^x 4^y = 16$$
$$\lg(4x - y) = \lg 2 + \lg 5$$

[4]

(b) Without using a calculator, simplify $\dfrac{\lg\sqrt{3} + \lg\sqrt{9} - \lg\sqrt{8}}{\lg 3 - \lg 2}$. [3]

(c) Using a suitable substitution, solve $4^x - 14\left(\frac{1}{4}\right)^x = 5$, giving your answer to 2 decimal places. [4]

End of Paper