BENDEMEER SECONDARY SCHOOL
2024 END OF YEAR EXAMINATION
SECONDARY THREE EXPRESS

CANDIDATE NAME	
CLASS	
INDEX NUMBER	

ADDITIONAL MATHEMATICS

4049

1 Oct 2024

2 hr 15 minutes

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
All the diagrams in this paper are **not** drawn to scale.
The use of an approved scientific calculator is expected, where appropriate.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR MARKER'S USE
90

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1 **(a)** Find the values of p for which $3x^2 + 2x = px - 3$ has equal roots. [3]

(b) **(i)** Given that $ax^2 - 5x + c$ is always positive, what conditions must be applied to the constants a and c? [3]

(b) **(ii)** Give an example of values a and c which satisfy the conditions found in **(b)(i)**. [2]

2 A curve has the equation $y = 2x^2 - 3x - 4$.

(i) Express $2x^2 - 3x - 4$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [2]

(ii) Using your answers in **(i)**, explain whether the curves $y = 2x^2 - 3x - 4$ and $y = -4 - (4x - 3)^2$ will intersect. [3]

3 **(a)** Solve the equation $x = 5\sqrt{2} + \frac{12}{x}$, where $x \neq 0$, giving your answer in its simplest surd form. [4]

(b) Solve the inequality $2x^2 - 3x > 9$ and represent the solution set on the number line. [3]

4 **(a)** **(i)** Find, in ascending powers of x, the first 3 terms in the expansion of $(1 - 3x)^9$. [2]

(ii) **Hence**, find the coefficient of x^2 in the expansion of $(2 + 3x)(1 - 3x)^9$. [2]

(b) In the binomial expansion of $\left(\sqrt{x} + \frac{h}{\sqrt{x}}\right)^{10}$, find in terms of h, [4]

(i) the term independent of x, and

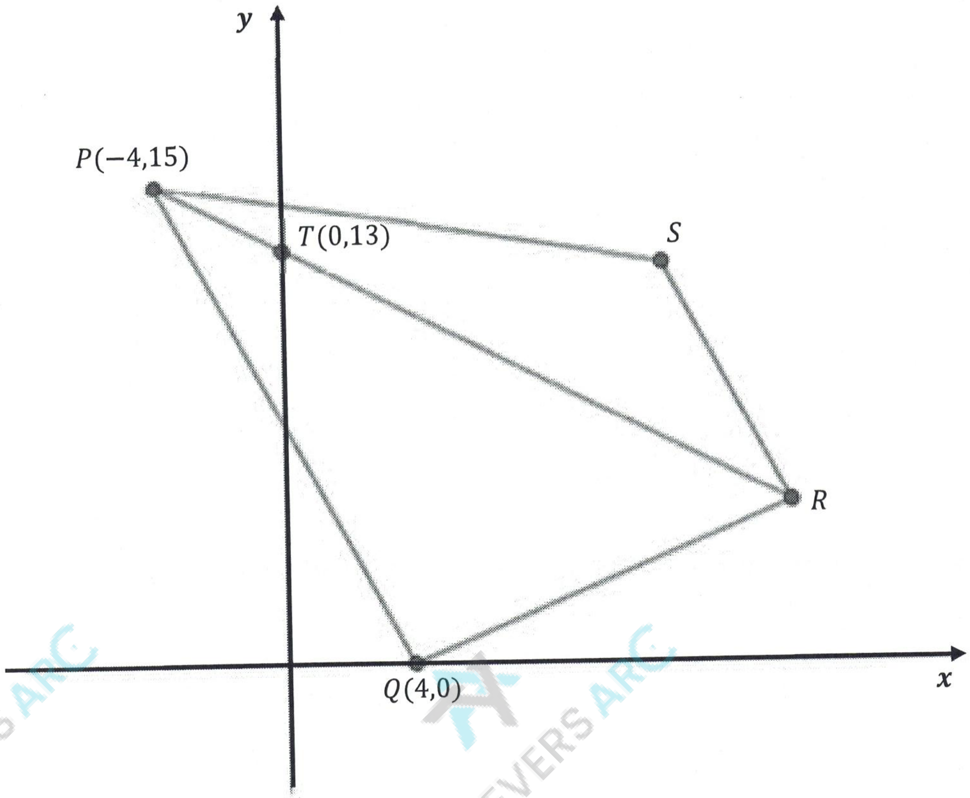
(ii) the coefficient of $\frac{1}{x}$.

5 **(a)** It is given that $f(x) = 2x^3 + ax^2 + bx - 60$, where a and b are constants, has a factor of $(x - 4)$ and leaves a remainder of -6 when divided by $(x + 2)$.
Find the value of a and of b. [4]

(b) Using the value of a and b from **(a)**, solve the equation $2x^3 + ax^2 + bx - 60 = 0$. [4]

(c) Hence or otherwise, solve the equation $2(x - 1)^3 + 3(x - 1)^2 - 29x - 31 = 0$. [3]

6 **Solutions to this question by accurate drawing will not be accepted.**
The diagram below shows a trapezium with vertices $P(-4,15)$, $Q(4,0)$, R and S.
The diagonal PR of the trapezium intersects the y −axis at $T(0,13)$, $PQ = 2SR$ and $QR = 13$ units.



(i) Find the equation of the line PR, and hence show that the coordinates of R are $(16,5)$. [5]

(ii) Is PQR a right-angled triangle? Explain and support your answer with working. [2]

(iii) Find the coordinates of S. [2]

(iv) Find the area of the trapezium. [2]

7 **(a)** **(i)** Given that $m = 2^x$, express $2^{2x-2} + k = 6(2^{x-2})$ as a quadratic equation in m. [3]

(ii) Find the range of values for k for which there are no real solutions for $2^{2x-2} + k = 6(2^{x-2})$. [2]

(b) **(i)** Prove that $7^{k+1} + 42(7^{k-1})$ is divisible by 13 for all positive integers of k. [2]

(ii) Given that $5(2^{3-2x}) = 6^x \times 2^{1-x}$, evaluate 12^x **without the use of a calculator**. [2]

8 Express $\frac{x^3+3x^2+2x+2}{x^3+2x}$ in partial fractions. [5]

9 **(a)** Given that $\sin\theta = -\frac{12}{13}$ and $180° < \theta < 270°$, find the following **without the use of a calculator.**

(i) $\cos(90° - \theta)$, [1]

(ii) $\tan\theta$. [2]

(b) **(i)** Solve the equation $\sin^2 x + \frac{1}{2} = 1$ for $0 < x < \pi$, giving your answers in terms of π. [3]

9 (b) (ii) Given that $\sin 55° = p$, express each of the following in terms of p.

(a) $\cos 35°$ [1]

(b) $\sin 305°$ [1]

(c) $\cos 125°$ [2]

10 **(a)** On a certain date, 150 cases of influenza were recorded in a town. This number increased with time and after t days, the number of cases was N. It is believed that N can be modelled by the formula $N = 150e^{pt}$. The number of cases recorded after 5 days was 220.

(i) Estimate the number of cases after 11 days. [4]

Influenza is declared an epidemic when the number of cases reach 400.

(ii) Estimate after how many days that the influenza is declared an epidemic. [2]

10 **(b)** Solve the equation $3\log_p 3 + 2\log_3 p = 7$. [4]

[1]

11 The table below shows experimental values of two variables x and y. It is known that x and y are related by an equation of the form $y = x^2 + ax - b$, where a and b are constants.

x	0.5	1	2	2.5	3	3.5
y	-3.75	-4	-3	-1.75	0	2.25

(i) Draw the graph of $(y - x^2)$ against x on the grid provided on page 17. [3]
Use the scale of 4 cm to 1 unit on the x −axis and 1 cm to 1 unit on the $(y - x^2)$ axis.

(ii) Use your graph to estimate the value of the constants a and b, [3]

End of Paper

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