BEDOK VIEW SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2024

CANDIDATE NAME			
REGISTER NUMBER		CLASS	

ADDITIONAL MATHEMATICS
Secondary 3 Express

4049

8 October 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

Setter : Mrs Kuek

This document consists of **21** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta ABC = \frac{1}{2}ab\sin C$$

1 Solve the simultaneous equations.

$$\frac{x}{3} - \frac{y}{4} = -2$$

$$2xy = 65$$

[4]

2 **(a)** Express $y = -2x^2 + 6x - 5$ in the form $y = a(x+b)^2 + c$ where a, b and c are constants. [3]

(b) Hence, write down the coordinates of the maximum point of y. [1]

(c) Sketch the graph of $y = -2x^2 + 6x - 5$, indicating the turning point, x and y intercept(s) if any. [2]

3 Solve $x\sqrt{7} = \sqrt{72} + x\sqrt{2}$ and express your answer in the form $\frac{a+b\sqrt{14}}{5}$, where a and b are integers. [4]

4 **(a)** Find the range of values of k for which $x^2 + 2kx - 3x + 3 - \frac{11}{4}k$ is always positive for all real values of x. [4]

(b) The line $y = p^2x + p$ is a tangent to the curve $y = px^2 + q^2x + q$, where $p \neq q$. Show that p and q are related by the equation $(q + p)^2(q - p) = 4p$. [5]

5 **(a)** Solve $\log_3 2 + 4\log_9 x = \log_3 (x-1) + 2$. [5]

(b) Solve $16^{x+1} = 8(4^x) + 15$ by using a suitable substitution. [4]

(c) Sketch the graph of $y = \log_5 x$, indicating all the critical point(s) if any. [1]

6 (a) (i) By considering the general term in the binomial expansion of $\left(px-\frac{1}{3x}\right)^{11}$, where p is a constant, explain why there is no even powers of x in this expansion. [3]

(ii) Find the values of p given that the coefficient of x in the expansion of $\left(px-\frac{1}{3x}\right)^{11}$ is $-121\frac{55}{81}$. [4]

Continuation of working space for **part (a)(ii)**

(b) Given that the sum of the coefficient of the second and third term of $\left(1+x^3\right)^n$ is 36, find the value of n. [3]

7 **(a)** Find the equation of the perpendicular bisector of AC, where the coordinates of A and C are (1, 2) and (7, 6) respectively. [4]

(b) B is a point on the y – axis such that B is equidistant from A and C. Explain why B is (0, 10). [1]

(c) Given that $ABCD$ is a rhombus, find the coordinates of D. [2]

(d) Find the area of the rhombus $ABCD$. [2]

(e) Find the perpendicular distance from A to BC. [3]

8 Express $\frac{2x^2 - 25x + 44}{x^2 - 9x + 8}$ in partial fractions. [5]

9 The expression $ax^3 - x^2 + bx + c$ is divisible by x and $x+2$ but leaves a remainder of -9 when divided by $(x-1)$.

(a) Find the value of a, of b and of c. [5]

(b) Hence factorise $ax^3 - x^2 + bx + c$ completely. [2]

10 **(a)** Solve $\sin 2x = \sin x$ for $0° < x < 180°$. [4]

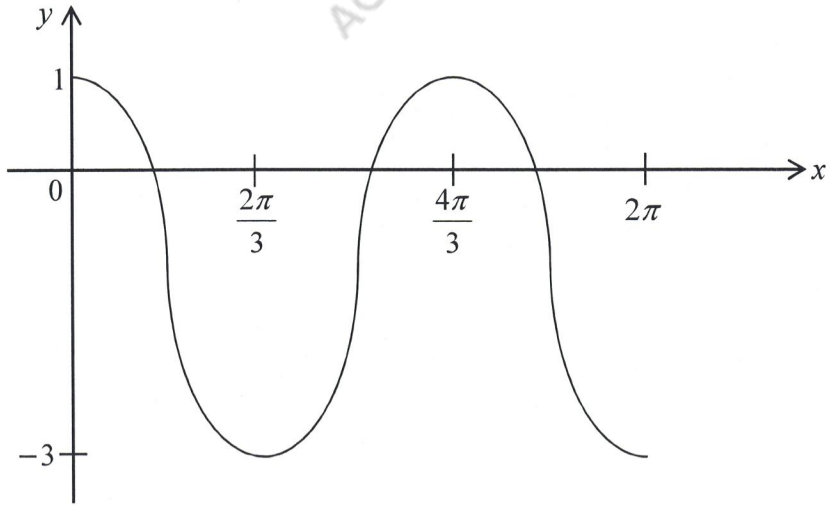
(b) Hence, find the corresponding value of y in $\cos^2(x+y) = 1$ for $0° < y < 180°$. [1]

11 **(a)** Given that $\cos\left(\theta-\frac{\pi}{3}\right)=2\cos\left(\theta+\frac{\pi}{3}\right)$, show that $\tan\theta=\frac{\sqrt{3}}{9}$. [4]

(b) Hence, solve $\cos\left(\theta-\frac{\pi}{3}\right)-2\cos\left(\theta+\frac{\pi}{3}\right)=0$ for $0<\theta<7$. [2]

12 **(a)** Prove that $\frac{\sec^2 x - 2\tan x}{1-\tan^2 x} = \frac{\cos x - \sin x}{\cos x + \sin x}$. [5]

(b) The graph below shows part of a curve $y = p\cos qx + r$ for $0 \le x \le 2\pi$.



Find the value of p, of q and of r. [3]

13 Given that θ is acute and $\sin\theta = q$, express each of the following in terms of q.

(a) $\text{cosec}\,\theta$ [1]

(b) $\tan(90° - \theta)$ [2]

(c) $\cos(-\theta)$ [1]

END OF PAPER