Name: .... (   )   Class: ....

ASSUMPTION ENGLISH SCHOOL
END-OF-YEAR EXAMINATION 2024

ADDITIONAL MATHEMATICS (4049/01)

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LEVEL: Sec 3 Express   **DATE:** 7 October 2024

CLASS: Sec 3/1, 3/3   **DURATION:** 1 Hour 45 Minutes

Additional Materials provided: NIL

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to 3 significant figures. Give answers in degrees to 1 decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

For Examiner's use:	
Total	/ 70

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

This Question Paper consists of 19 printed pages and 1 blank page

[Turn over]

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 Determine the nature of the roots to the equation $x^2+(p+2)x+2p=0$, where p is a constant and $p \neq 2$. [3]

2 Solve the simultaneous equations

$$2y = x - 3$$
$$\frac{20}{y} = x + 3$$

[4]

3 Determine the x-intercept(s) of the curve $y = 3^{2x+1} - 2(3^{x}) - 5$. [4]

4 **(a)** The polynomial $P(x) = 2x^3 - ax^2 - x + 1$, where a is a constant, leaves a remainder of 2 when divided by $x - 1$. Find the value of a. [1]

(b) Hence, solve the equation $P(x) = 0$. [4]

5 It is given that $\lg(x^2 y) = p$ and $\lg\left(\frac{x^3}{\sqrt{y}}\right) = q$, where p and q are constants.

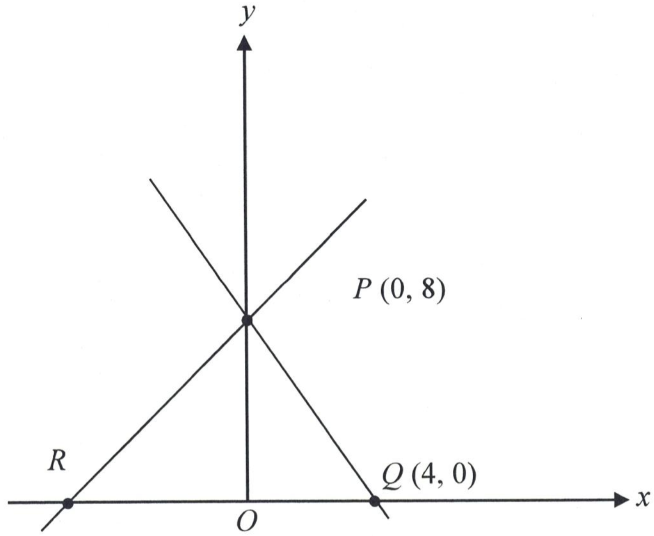
(a) Show that $2\lg x + \lg y = p$. [2]

(b) Write down a similar equation relating $\lg x$ and $\lg y$. [1]

(c) Hence, or otherwise, express $\lg x$ in terms of p and q. [2]

TURN OVER FOR QUESTION 6

6 The diagram shows two straight lines PQ and PR. The coordinates of P and Q are (0, 8) and (4, 0) respectively. R is a point on the x-axis.



(a) Given that the line PR makes an angle of inclination of 45° with the positive x-axis, show that the equation of the line PR is $y = x + 8$. [2]

(b) Point S lies on the line PR extended such that S is equidistant from P and Q. Find the coordinates of S. [4]

(Continuation of working space for Question 6)

7 (a) Solve the equation $\sec\theta = \cos\theta$ for $-270° < \theta < 0°$. [3]

(b) Solve the equation $\cos(2x)+7\sin x-4=0$ for $0\leq x\leq 4\pi$, leaving your answers in terms of π. [4]

8 **(a)** In the expansion of $\left(a-\frac{x^2}{k}\right)^{10}$ where a and k are positive integers, the coefficient of x^4 is 1280, and the term independent of x is 1024.
Find the values of a and of k. [5]

(b) Using the values of a and k in **(a)**, find the coefficient of x^6 in the expansion $\left(1-\frac{1}{x^4}\right)\left(a-\frac{x^2}{k}\right)^{10}$. [3]

9 **(a)** Explain why $x^3 - x^2 + 2x - 2 = 0$ has only one real root. [2]

(b) Hence, express $\dfrac{x^4 + 2x^2 + 2x - 2}{x^3 - x^2 + 2x - 2}$ in partial fractions. [6]

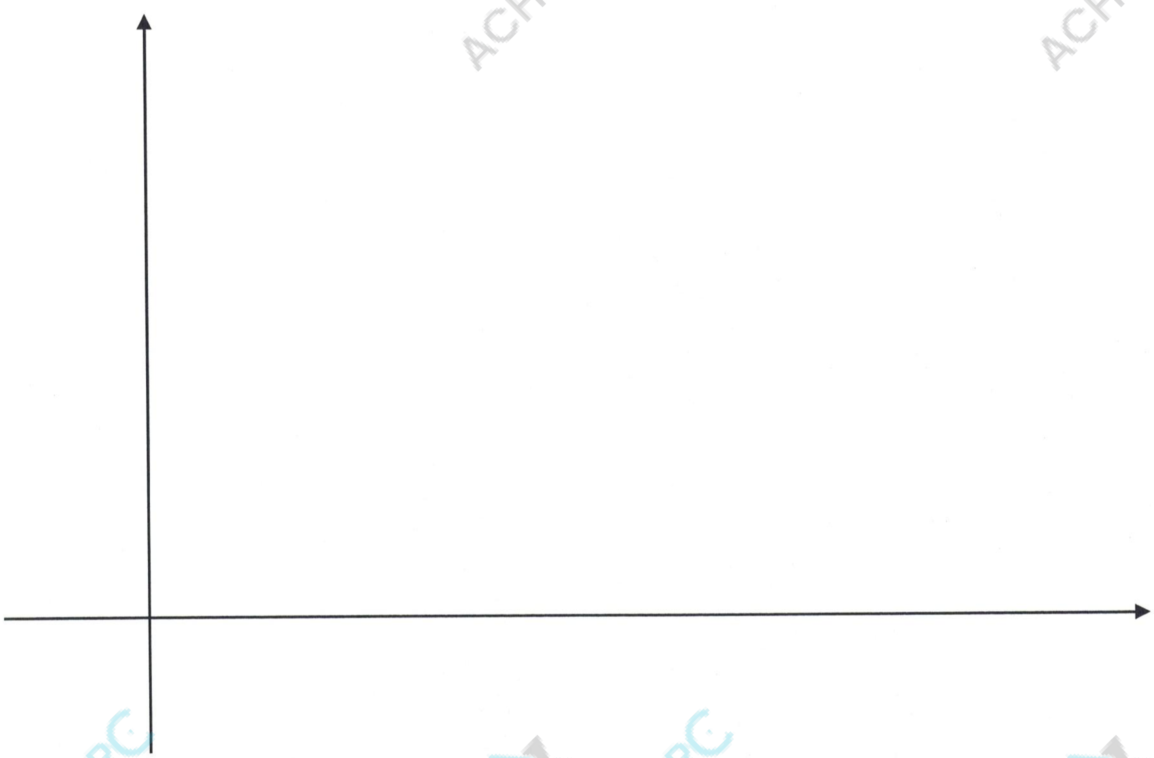
(Continuation of working space for Question 9)

10 The curve $y = a\sin bx + c$ is defined for $0 \leq x \leq 2\pi$, where a, b and c are positive constants. It is given that the maximum and minimum values of y are 5 and 1 respectively.

(a) Explain why $a = 2$ and $c = 3$. [2]

(b) It is given that the curve has consecutive maximum points at $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$. Explain why $b = 2$. [2]

(c) Sketch the graph of $y = a\sin bx + c$ for $0 \leq x \leq 2\pi$ on the axes below, indicating clearly the coordinates of all maximum and minimum points. [3]



(d) The equation $a\sin bx + c = d$ has exactly two solutions for $0 \le x \le 2\pi$. State the possible values of d. [2]

11 **(a)** It is given that $\sin 24° = k$, where k is a constant. Without the use of a calculator, express each of the following in terms of k.

(i) $\cos 24°$, [1]

(ii) $\tan 156°$, [2]

(iii) $\sin 48°$. [2]

(b) **(i)** Prove the identity $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$. [4]

(ii) Hence, or otherwise, show that the equation $\tan 3x = 11\tan x$ can be reduced to $A\tan^3 x + B\tan^2 x + C\tan x + D = 0$, where A, B, C and D are constants. [2]

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Answer **all** the questions.

1 Find the range of values of m for which the two curves $y=\frac{3}{x^2}$ and $y=\frac{k}{x}-3$ have exactly two different points of intersection. [4]

2 If $a=\frac{1+\sqrt{5}}{2}$ and $b=\frac{1-\sqrt{5}}{2}$, show that

(a) $a-1=\frac{1}{a}$, [3]

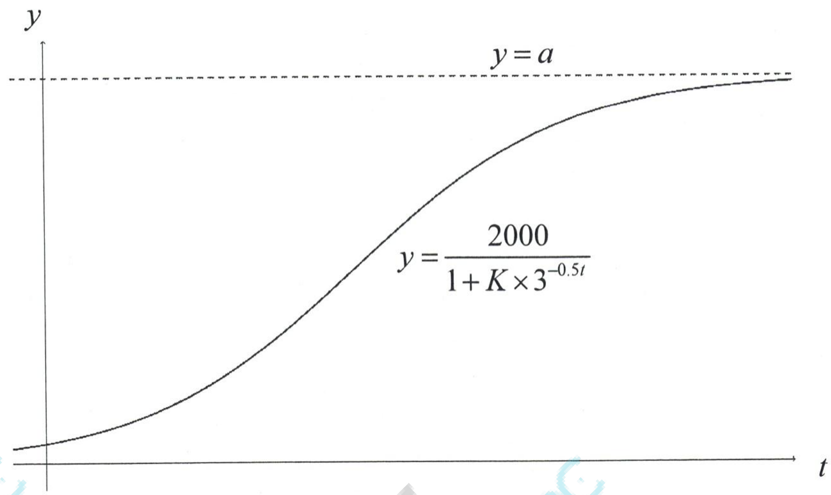
(b) a^2+b^2 is a rational number. [2]

3 Solve the simultaneous equations

$$\left(9^x\right)\left(27^y\right) = 3$$
$$8^x = 16\left(4^{-y}\right)$$

[5]

4 In a small town of 2000 people, there was an outbreak of a contagious virus. The virus spread across the town population is modelled by the equation $y = \dfrac{2000}{1 + K \times 3^{-0.5t}}$, where y is the number of people infected after t days, and a and K are constants. The graph of this curve is shown below.



(a) The curve has an asymptote at $y = a$. Explain using the equation of the curve, why $a = 2000$. [2]

(b) The infection started from a single person. Explain why $K = 1999$. [2]

(c) A quarantine order will be issued when at least 30% of the population have been infected. After how many days will such an order be issued? [3]

5 **(a)** Find the range of values of k for which $y = 2x^2 + kx + 5 - \frac{3}{8}k$ is always positive for all real values of x. [3]

(b) For $k = -1$, express $y = 2x^2 + x - 8k - 3$ in the form $y = a(x+b)^2 + c$. State the coordinates of the turning point $y = 2x^2 + x - 8k - 3$. [3]

6 **(a)** Solve the equation $\lg(x+1)-\lg(x-1)=\log_3 9$. [3]

(b) Solve the equation $\log_3 x^2 - 1 = 3\log_x 3$, leaving your answer(s) in exact form. [5]

7 **(a)** Two acute angles A and B are such that $\cos(A+B)=-\frac{4}{5}$, and $\sin A \sin B = \frac{14}{15}$. Find the values of

(i) $\cos A \cos B$, [2]

(ii) $\tan A \tan B$. [2]

(b) Prove the identity $(\cot B - 1)\cot\left(\frac{\pi}{4} - B\right) = 1 + \cot B$. [4]

8 **(a)** Show that $\dfrac{3\sin x - \cos 2x - 1}{\sin^2 x + 2\sin x} = 2 - \operatorname{cosec} x$ [4]

(b) Hence, or otherwise, solve the equation $\dfrac{3\sin x - \cos 2x - 1}{\sin^2 x + 2\sin x} = 4\cot^2 x - 8$ for $0° \le x \le 360°$. [5]

9 The table below shows some experimental values of two variables, R and t.

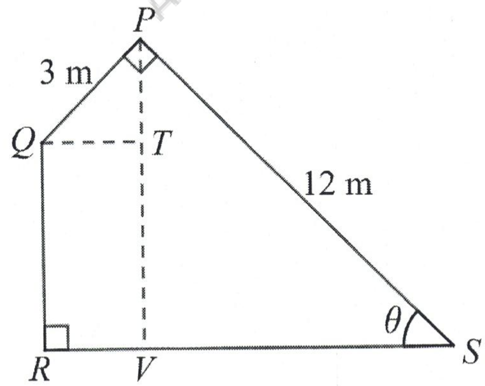
t	2	5	9	17	22	28
R	9.72	2.65	1.15	0.47	0.32	0.23

It is known that the two variables R and t are related by the equation $R = At^{-b}$, where A and b are constants.

(a) Plot $\lg R$ against $\lg t$ for the given data and draw a straight line graph. [3]

(b) Using your graph, estimate the value of A and of b. [4]

10



The diagram shows a quadrilateral $PQRS$ in which angle QPS = angle $QRS = 90^\circ$, angle $PSR = \theta°$, $PQ = 3$ m and $PS = 12$ m.
The points T and V are such that PV is perpendicular to both QT and RS.

(a) Show that $QT = 3\sin\theta$. [2]

(b) Find, in terms of θ, the length of TV. Hence, show that the perimeter of the quadrilateral $PQRS$, L m, is given by $L = 15\sin\theta + 9\cos\theta + 15$. [4]

(c) Express L in the form of $R\sin(\theta+\alpha)+k$, where $R>0$, $0<\alpha<90^{\circ}$ and k is a constant. [3]

(d) Hence, find the maximum value of L, and sate the corresponding value of θ when this occurs. [2]

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