Candidate Name	Form Class	Index Number

ANG MO KIO SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2024
SECONDARY THREE EXPRESS

ADDITIONAL MATHEMATICS

4049
08 October 2024
2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is **90**.

For Examiner's Use
90

This document consists of **23** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

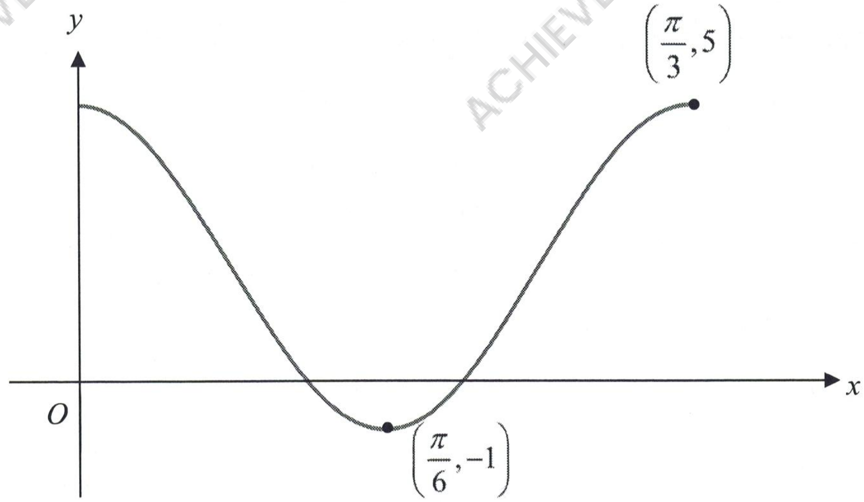
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Find the value of n for which the division of $2x^n + 4x^2 - 10x - 20$ by $x - 2$ gives a remainder of 40. [3]

2 By using suitable substitution, solve $5^{1+2x} - 6\left(5^{x}\right) + 1 = 0$. [4]

3



The diagram shows the curve $y = p + q\cos rx$ for $0 \le x \le \frac{\pi}{3}$. The curve has a minimum point at $\left(\frac{\pi}{6}, -1\right)$ and a maximum point at $\left(\frac{\pi}{3}, 5\right)$.

(a) Find the value of each of the constants p, q and r. [3]

(b) State the number of solutions of the equation $p + q\cos rx = 0$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [1]

4 **(a)** Using factor theorem, show that the polynomial $\mathrm{P}(x) = x^3 - 7x^2 + 2x + 40$ is divisible by $x^2 - 2x - 8$. [2]

(b) Factorise $\mathrm{P}(x)$ completely. [3]

(c) Hence solve the equation $P(x) = 3(x+2)(x-4)$. [2]

5 Given that $\sin A = \frac{5}{13}$ where $\frac{\pi}{2} < A < \pi$ and $\cos B = \frac{3}{5}$ where $\pi < B < 2\pi$, without using a calculator, find the value of

(a) $\sec A$, [2]

(b) $\cos(A - B)$, [2]

(c) $\cos\frac{B}{2}$. [2]

6 **(a)** Show that the curve $x^2 - 3px + 3p - 1 = 0$ has real roots for all values of p. [3]

(b) Find the range of values of q for which the curve $y = x^2 - 2(q-3)x + 25$ lies entirely above the x–axis. [4]

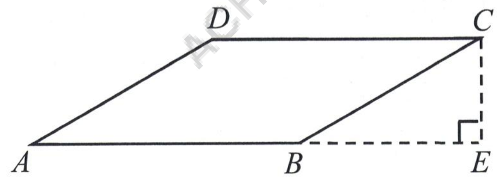
7 In a Physics lesson, a tennis ball launcher was built. The projectile motion of the tennis ball can be modelled by the equation $h = 2t - \frac{1}{3}t^2 + 1$, where height, h metres, of the projectile above the ground is related to t, the time in seconds for the motion.

(a) Express $2t - \frac{1}{3}t^2 + 1$ in the form of $a(t+b)^2 + c$, where a, b and c are constants to be determined. [3]

(b) Sketch the graph $h = 2t - \frac{1}{3}t^2 + 1$ for $h \geq 0$. [2]

(c) Show that the tennis ball is at least 2 metres above the ground for more than half of its motion time. [3]

8

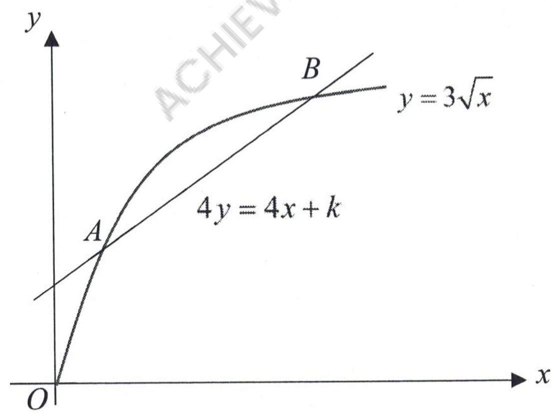


The diagram shows a parallelogram *ABCD* whose area is $\left(5+4\sqrt{2}\right)\text{cm}^2$. The line *CE* is perpendicular to the line *ABE*. The length of *AE* is $\left(1+5\sqrt{2}\right)$ cm and the length of *AB* is $\left(3+\sqrt{2}\right)$ cm. Express in the form $p+q\sqrt{2}$, where p and q are integers,

(a) the length of *CE*, [4]

(b) the area of triangle *BEC*. [3]

9



The diagram shows the curve $y=3\sqrt{x}$ and the line $4y=4x+k$ where k is a constant.

The line intersects the curve at points A and B.

(a) In the case where the line is a tangent to the curve, find the value of k. [4]

(b) In the case where $k = 8$, find the coordinates of A and B. [4]

10 Express $\dfrac{2x-7}{(x+4)(x-1)^2}$ in partial fractions. [5]

11 **(a)** Without the use of a calculator, find the value of $\log_8 m \times \log_m 2$. [2]

(b) Given that $\log_2(2y-4)-\log_2(x+1)=4$, express y in terms of x. [3]

12 **(a)** Prove that $(\sec x + 1)(\operatorname{cosec} x - \cot x) = \tan x$. [4]

(b) **Hence** find the values of x between $0°$ and $360°$ which satisfy the equation

$$2\tan^2 x - 5 = 3(\sec x + 1)(\operatorname{cosec} x - \cot x).$$

[5]

13 **(a)** **(i)** Find the first 3 terms in the expansion of $(1+2x)^7$. [2]

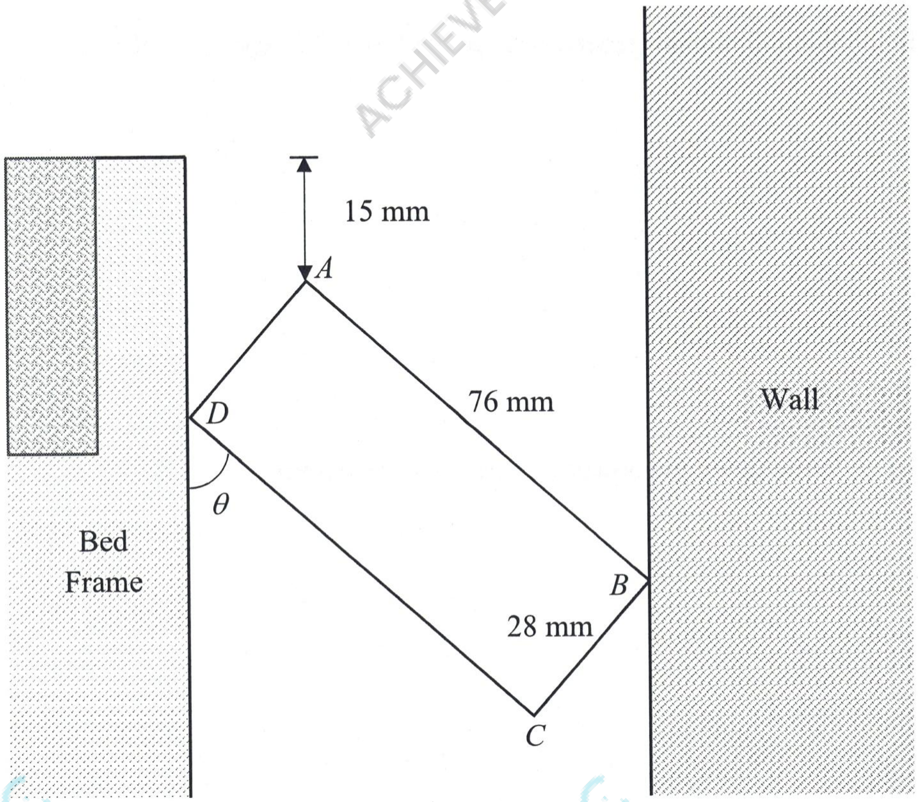
(ii) Hence find the coefficient of x^2 in the expansion of $(1-3x)^2(1+2x)^7$. [2]

(b) **(i)** Write down the general term in the binomial expansion of $\left(2x^2 - \frac{1}{x}\right)^{13}$. [1]

(ii) Write down the power of x in this general term. [1]

(iii) Hence explain if there is a term in x in the binomial expansion of $\left(2x^2 - \frac{1}{x}\right)^{13}$. [2]

14



The diagram shows the cross-sectional view of a rectangular box *ABCD* stuck in the gap between a vertical bed frame and wall. The width of the box, *AB*, is 76 mm and its depth, *BC*, is 28 mm. Point *A* is 15 mm from the top of the bed frame.

(a) Show that *H* mm, the vertical distance of point *C* from the top of the bed frame, is given by $H = 15 + 76\cos\theta + 28\sin\theta$. [2]

(b) Express H in the form $p + R\cos(\theta - \alpha)$, where p is a constant, $R > 0$ and $0° \le \alpha \le 90°$. [4]

(c) Find the value(s) of θ when $H = 92$ mm. [3]

END OF PAPER

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